Reduction of the Dispersion Error in the Interpolated Digital Waveguide Mesh Using Frequency Warping

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Introduction

- **Digital waveguides** are useful in physical modeling of musical instruments and other acoustic systems (Smith, 1992, 1997)

- **2-D digital waveguide mesh** (WGM) for simulation of plates, drums etc. (Van Duyne & Smith, 1993)

- **3-D digital waveguide mesh** for simulation of acoustic spaces (Savioja *et al.*, 1994)

- 3-D simulation also using wave digital filters (Schetelig & Rabenstein, ICASSP’98)
Sophisticated Waveguide Structures

- In the original WGM, wave propagation speed depends on direction and frequency (Van Duyne & Smith, 1993)
- More advanced structures improve this problem, e.g.,
  - triangular WGM (Van Duyne & Smith, 1995, 1996; Fontana & Rocchesso, 1998)
  - interpolated WGM (Savioja & Välimäki, ICASSP’97)
- Direction-dependence is reduced but frequency-dependence remains
  ⇒ Dispersion!
Interpolated 2-D Waveguide Mesh

Original WGM

Hypothetical 8-directional WGM

Interpolated WGM

(Van Duyne & Smith, 1993)

(Savioja & Välimäki, ICASSP’97)
Wave Propagation Speed

Original WGM

Interpolated WGM (bilinear interpolation)
Relative Frequency Error (RFE)

RFE in diagonal and axial directions:

(a) original and
(b) interpolated rectangular WGM
Frequency Warping

- Dispersion error of the interpolated WGM can be reduced by frequency warping because
  - the difference between the max and min errors is small
  - the RFE curve is smooth

- Postprocess the response of the WGM using a warped-FIR filter (Oppenheim et al., 1971; Laine et al., ICASSP’94; Karjalainen & Smith, 1996)
Frequency Warping: Warped-FIR Filter

- Chain of first-order allpass filters

\[ A(z) = \frac{z^{-1} + \lambda}{1 + \lambda z^{-1}} \]

- \( s(n) \) is the signal to be warped
- \( s_w(n) \) is the warped signal
- The extent of warping is determined by \( \lambda \)

\( \delta(n) \rightarrow A(z) \rightarrow A(z) \rightarrow \ldots \rightarrow A(z) \rightarrow s_w(n) \)
Optimization of Warping Factor $\lambda$

- Different optimization strategies can be used, such as:
  - least squares
  - minimize maximal error (minimax)
  - maximize the bandwidth of $X\%$ error tolerance

- We present two results:
  (a) minimax
  (b) maximize 1%-error bandwidth
Reduced Relative Frequency Error

RFE in axial and diagonal directions after warping

(a) minimax: 
\[ \lambda = -0.1947 \]

(b) max 1%-error bandwidth: 
\[ \lambda = -0.1757 \]
Comparison

- Computational complexity:
  - Original WGM: 1 binary shift & 4 additions
  - Interpolated WGM: 3 MUL & 9 ADD
  - Warped-FIR filter: $O(L^2)$ where $L$ is the signal length
- 1%-error-bandwidth of the original WGM is $0.077f_s$ while that of the new algorithm is $0.22f_s$ → the bandwidth is 2.9 times wider!
- Original WGM requires 3 times more time steps and over 8 times more memory for the same bandwidth
Simulation Result vs. Analytical Solution

Eigenfrequencies of a square plate:
(a) original, 
(b) interpolated, and 
(c) warped interpolated 
($\lambda = -0.1757$) 
digital waveguide mesh
Error in Mode Frequencies

Error in eigenfrequencies of the square plate:

- Warped Interp. WGM \((\lambda = -0.1757)\)
- Original WGM
- Interpolated WGM
Warped Triangular Waveguide Mesh

- We have also applied the frequency-warping technique to the triangular WGM
- Results were published in the March 1999 issue of the *IEEE Signal Processing Letters*
- The warped triangular WGM is better than the interpolated rectangular one
- A “drawback” is the triangular tessellation
Conclusions and Future Work

- Accuracy of 2-D digital waveguide mesh simulations can be improved using
  1) the interpolated or triangular WGM and
  2) frequency warping
- The frequency error caused by dispersion can be reduced dramatically
- In the future, the warping technique will be applied to 3-D waveguide mesh simulations