Reduction of the Dispersion Error in the Interpolated Digital Waveguide Mesh Using Frequency Warping

Lauri Savioja and Vesa Välimäki

Helsinki University of Technology

Laboratory of Acoustics and Audio Signal Processing

(Espoo, Finland)

http://www.acoustics.hut.fi/

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Outline

- Introduction
- Interpolated 2-D Digital Waveguide Mesh
- Frequency Warping
- Simulation Example
- Conclusions

Introduction

- Digital waveguides are useful in physical modeling of musical instruments and other acoustic systems (Smith, 1992, 1997)
- 2-D digital waveguide mesh (WGM) for simulation of plates, drums etc. (Van Duyne & Smith, 1993)
- 3-D digital waveguide mesh for simulation of acoustic spaces (Savioja *et al.*, 1994)
- 3-D simulation also using wave digital filters (Schetelig & Rabenstein, ICASSP'98)

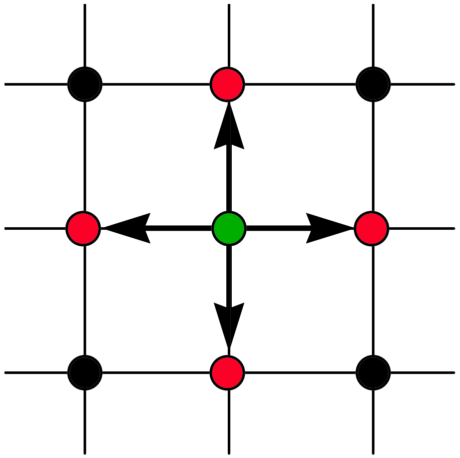
Sophisticated Waveguide Structures

- In the original WGM, wave propagation speed depends on direction and frequency (Van Duyne & Smith, 1993)
- More advanced structures improve this problem, e.g.,
  - triangular WGM (Van Duyne & Smith, 1995, 1996; Fontana & Rocchesso, 1998)
  - interpolated WGM (Savioja & Välimäki, ICASSP'97)
- Direction-dependence is reduced but frequency-dependence remains

  ⇒ Dispersion !

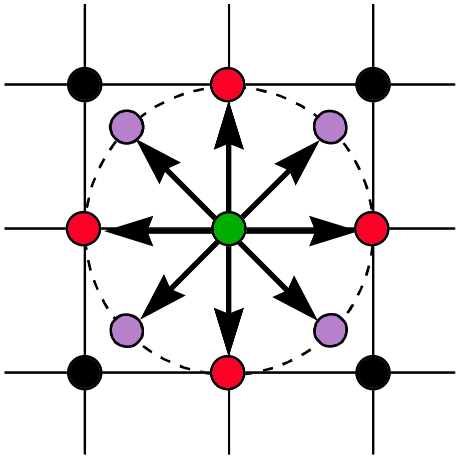
Interpolated 2-D Waveguide Mesh

Original WGM

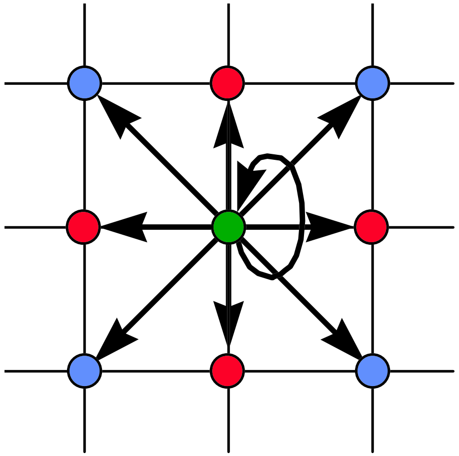


(Van Duyne & Smith, 1993)

Hypothetical 8-directional WGM



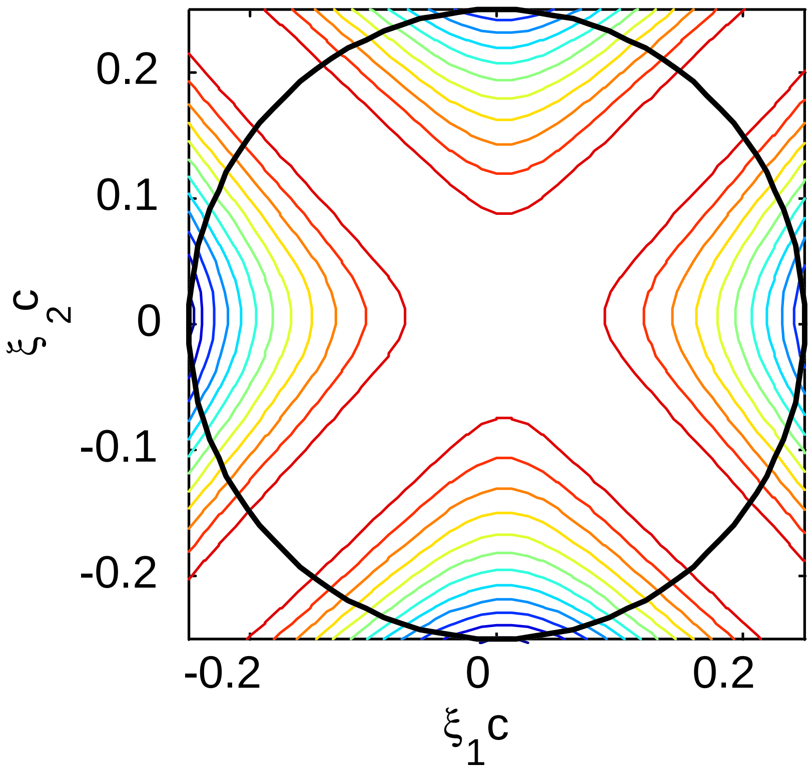
Interpolated WGM



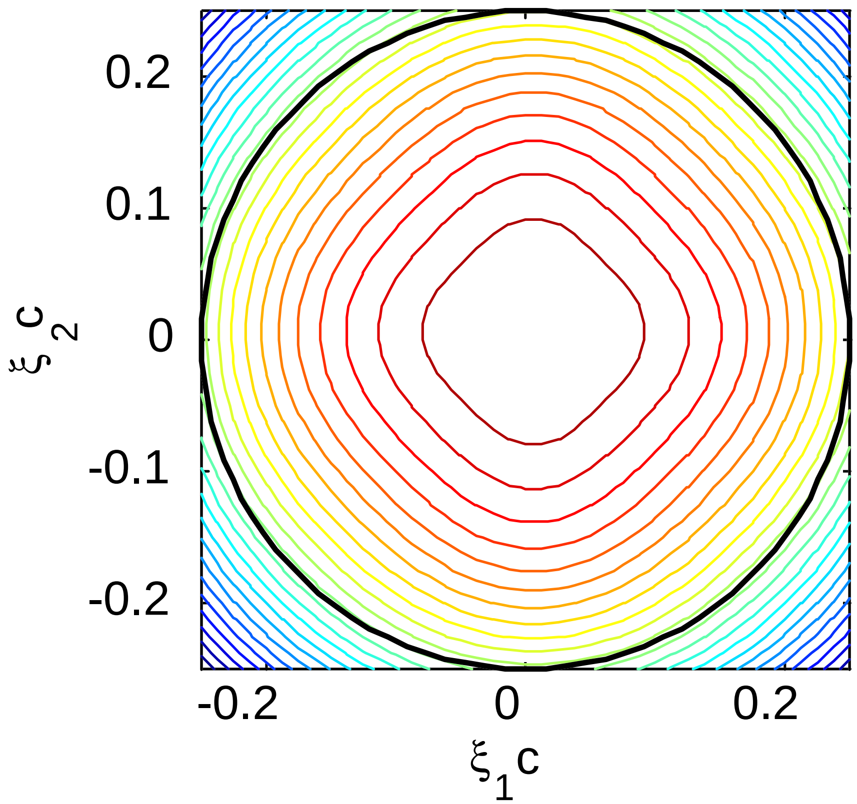
(Savioja & Välimäki, ICASSP'97)

Wave Propagation Speed

Original WGM



Interpolated WGM (bilinear interpolation)



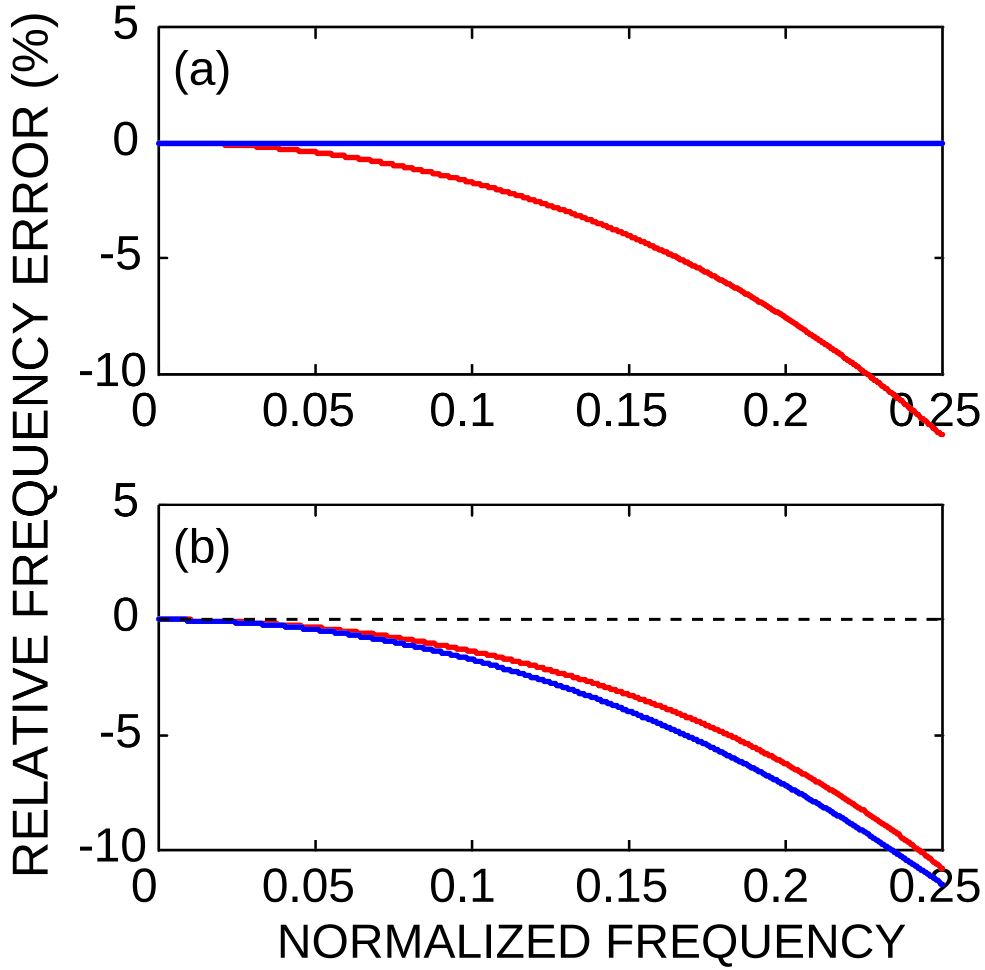
Relative Frequency Error (RFE)

RFE in diagonal and axial directions:

(a) original and

(b) interpolated

rectangular WGM



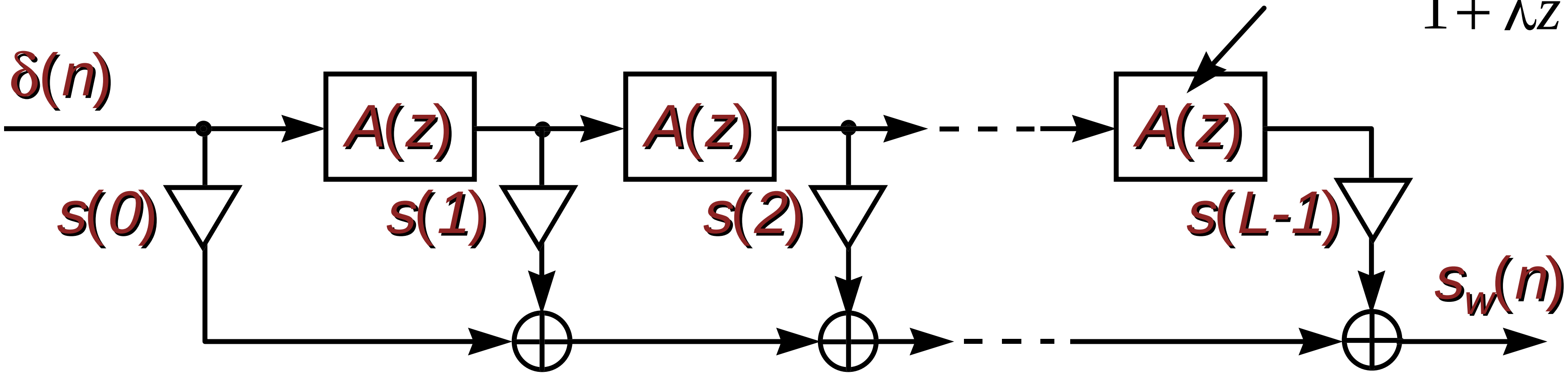
Frequency Warping

- Dispersion error of the interpolated WGM can be reduced by frequency warping because
  - the difference between the max and min errors is small
  - the RFE curve is smooth
- Postprocess the response of the WGM using a warped-FIR filter (Oppenheim *et al.*, 1971; Laine *et al.*, ICASSP'94; Karjalainen & Smith, 1996)

Frequency Warping: Warped-FIR Filter

- Chain of first-order allpass filters

$$A(z) = \frac{z^{-1} + \lambda}{1 + \lambda z^{-1}}$$



- $s(n)$ is the signal to be warped
- $s_w(n)$ is the warped signal
- The extent of warping is determined by λ

Optimization of Warping Factor λ

- Different optimization strategies can be used, such as:
  - least squares
  - minimize maximal error (minimax)
  - maximize the bandwidth of X% error tolerance
- We present two results:
  - (a) minimax
  - (b) maximize 1%-error bandwidth

Reduced Relative Frequency Error

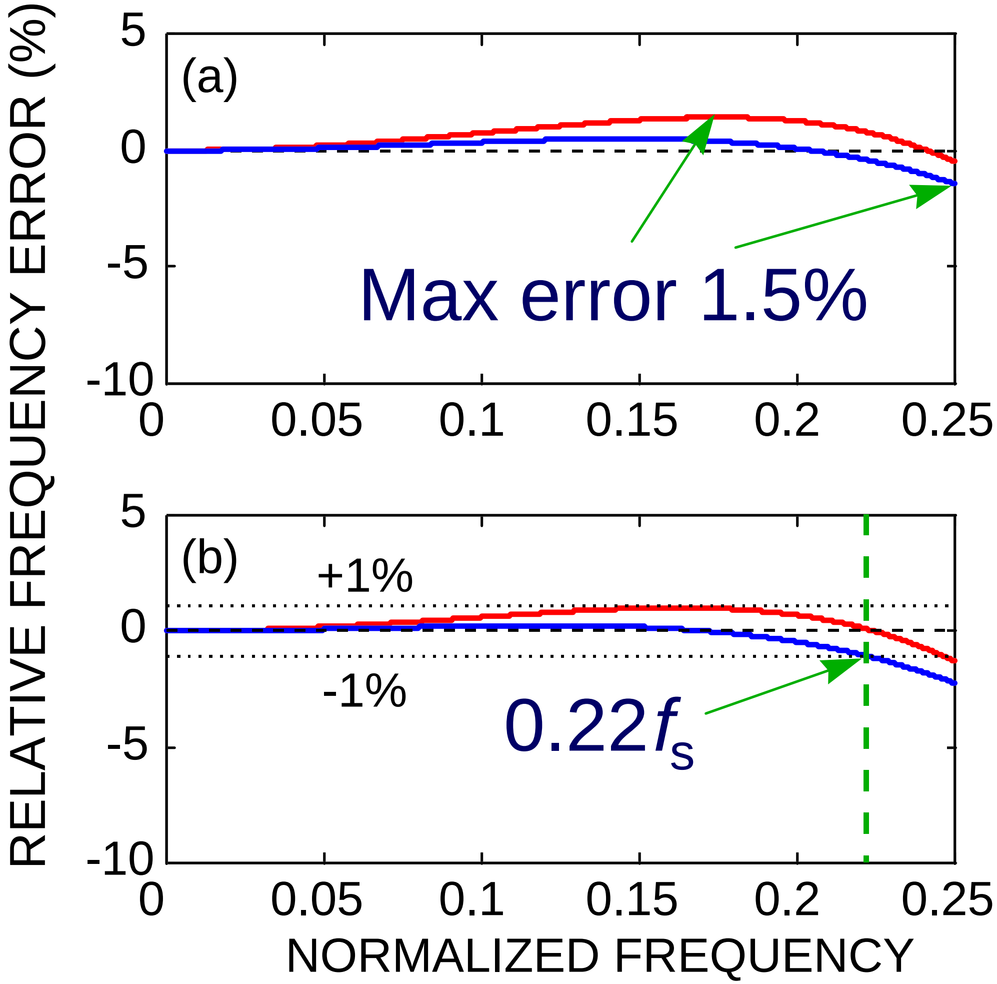
RFE in axial and diagonal directions after warping

(a) minimax:

$\lambda = -0.1947$

(b) max 1%-error bandwidth:

$\lambda = -0.1757$



Savioja and Välimäki 1999

Comparison

- Computational complexity:
  - Original WGM: 1 binary shift & 4 additions
  - Interpolated WGM: 3 MUL & 9 ADD
  - Warped-FIR filter: $O(L^2)$ where L is the signal length
- 1%-error-bandwidth of the original WGM is $0.077f_s$ while that of the new algorithm is $0.22f_s$

  $\Rightarrow$ the bandwidth is 2.9 times wider!
- Original WGM requires 3 times more time steps and over 8 times more memory for the same bandwidth

Simulation Result vs. Analytical Solution

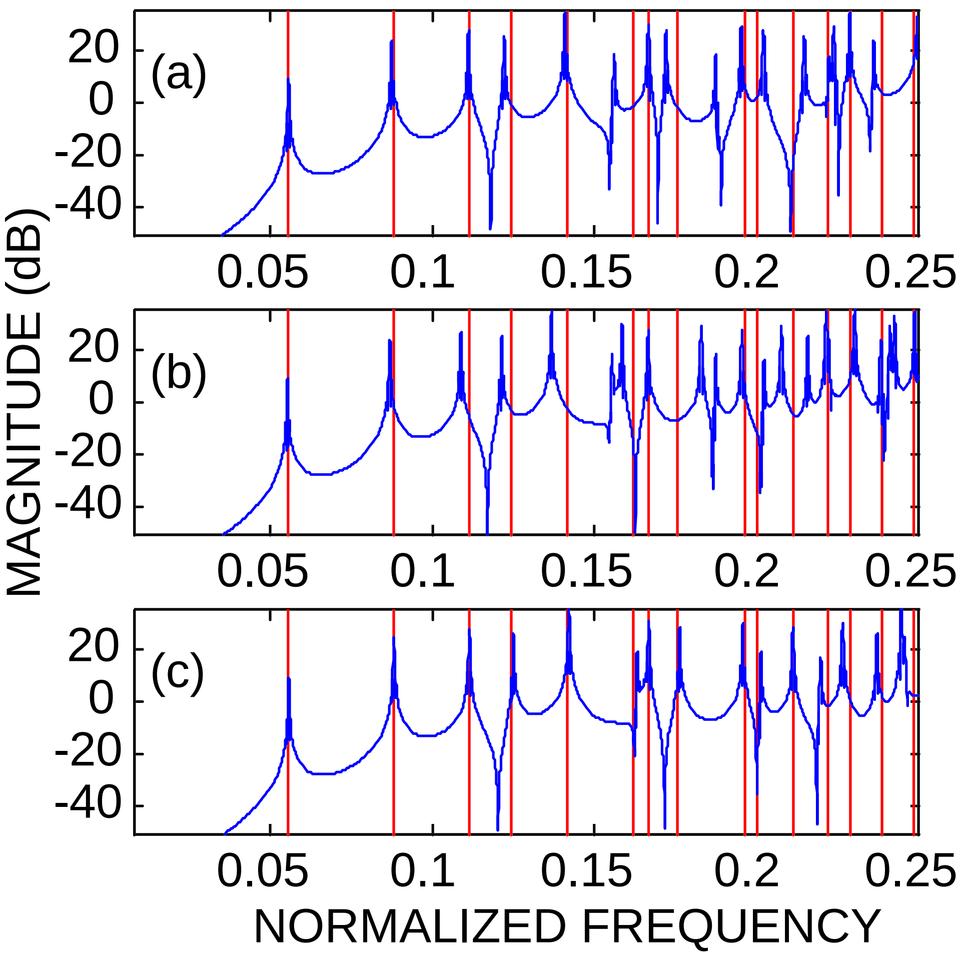
Eigenfrequencies of a square plate:

(a) original,

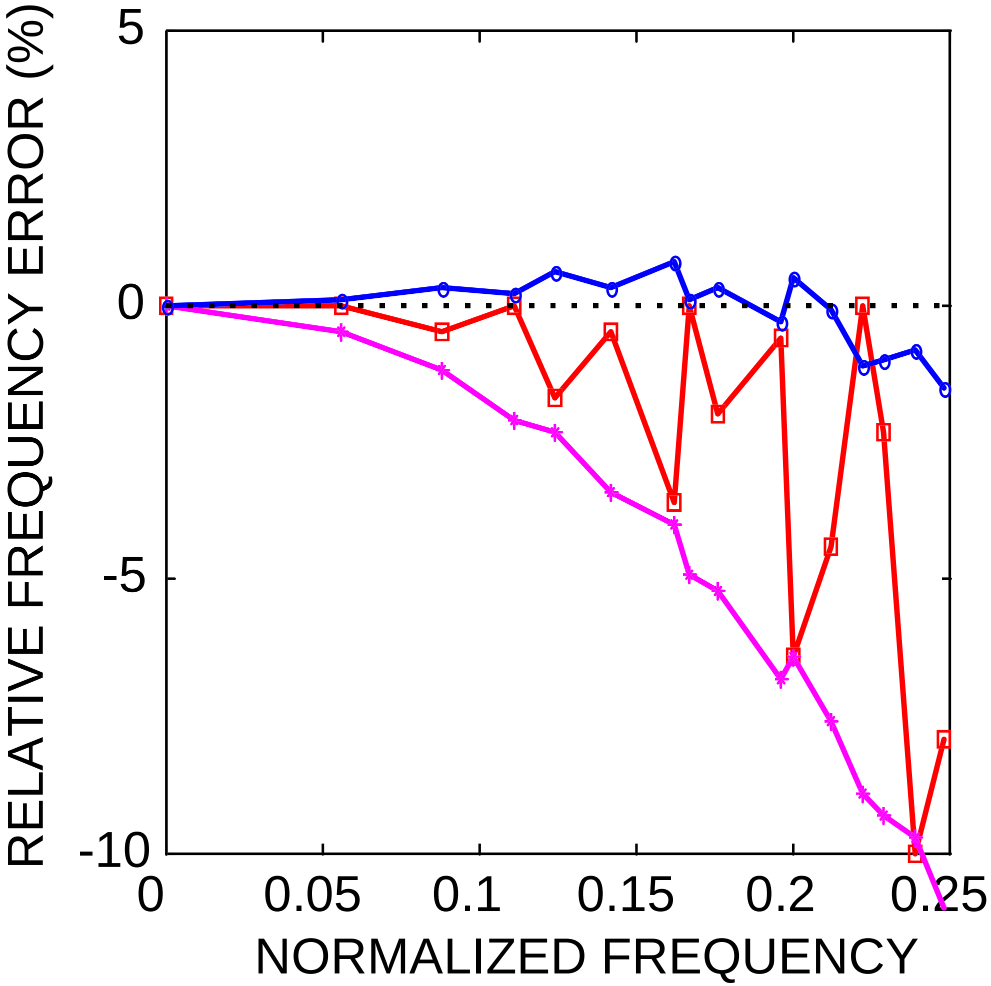
(b) interpolated, and

(c) warped interpolated (λ = -0.1757)

digital waveguide mesh



Error in Mode Frequencies



Error in eigenfrequencies of the square plate:

- Warped Interp. WGM (λ = -0.1757)
- Original WGM
- Interpolated WGM

Warped Triangular Waveguide Mesh

- We have also applied the frequency-warping technique to the triangular WGM
- Results were published in the March 1999 issue of the *IEEE Signal Processing Letters*
- The warped triangular WGM is better than the interpolated rectangular one
- A “drawback” is the triangular tessellation

Conclusions and Future Work

- Accuracy of 2-D digital waveguide mesh simulations can be improved using
  1) the interpolated or triangular WGM and
  2) frequency warping
- The frequency error caused by dispersion can be reduced dramatically
- In the future, the warping technique will be applied to 3-D waveguide mesh simulations