Principles of Fractional Delay Filters

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Principles of Fractional Delay Filters

1. Motivation
2. Ideal FD Filter and Its Approximations
3. FD Filters for Very Small Delay
4. Time-Varying FD Filters
5. Resampling of Nonuniformly Sampled Signals
6. Conclusions

1. Motivation: The Importance of Sampling at the Right Time

a) Uniform sampling problems

- Fine-tune sampling rate and/or instant

1) Constant delay: accurate time delays

2) Time-varying delay: resampling on a nonuniform grid

b) Nonuniform sampling problems

- Sampling instants determined, e.g., by physical constraints
- Resample on a uniform grid

1. Motivation: Many Applications (2)

- Sampling rate conversion
  - Especially conversion between incommensurate rates, e.g., between standard audio sample rates 48 and 44.1 kHz
- Music synthesis using digital waveguides
  - Comb filters using fractional-length delay lines
- Doppler effect in virtual reality
- Synchronization of digital modems
- Speech coding and synthesis
- Beamforming
- etc.

2. Ideal FD Filter and Approximations

- FD filter = digital version of a continuous time delay
- An ideal lowpass filter with a time shift: Impulse response is a sampled and shifted **sinc function**:

$$\mathrm{sinc}(n - D) = \sin[p(n - D)]/p(n - D)$$

where n is the time index; D is delay in samples

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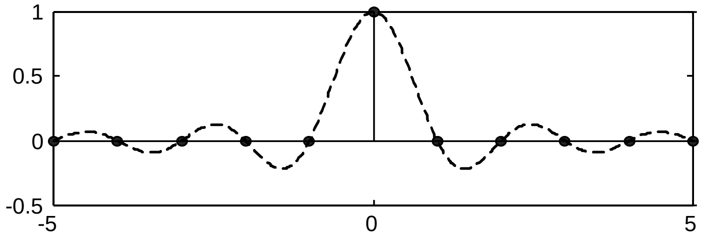
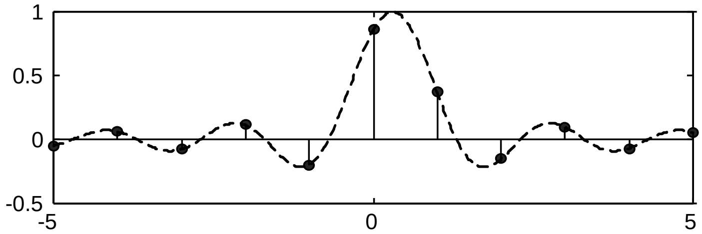
2. Ideal FD Filter and Approximations (2)

Sampled Sinc Function (D = 0)



Sampled & Shifted Sinc (D = 0.3)



Time in Samples

When D integer:

Sampled at zero-crossings

(no fractional delay)

When D non-integer:

Sampled between zero-crossings

⇒ Infinite-length impulse response

2. FIR FD Approximations

- FD must be approximated using FIR or IIR filters (see, e.g., Laakso *et al., IEEE SP Magazine*, 1996)
- FIR FD filters have **asymmetric impulse response** but they aim at having linear phase
- Approximation of complex-valued frequency response (magnitude and phase)
  $\Rightarrow$ traditional linear-phase methods not applicable
- Most popular technique: **Lagrange interpolation**

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2. Lagrange Interpolation

- Polynomial curve fitting = max. flat approximation
- Closed-form formula for coefficients:

$$h(n) = \prod_{\substack{k=0 \\ k \neq n}}^{N} \frac{D-k}{n-k} \quad \text{for} \;\; n = 0,\ 1,\ 2, \ldots,\ N$$

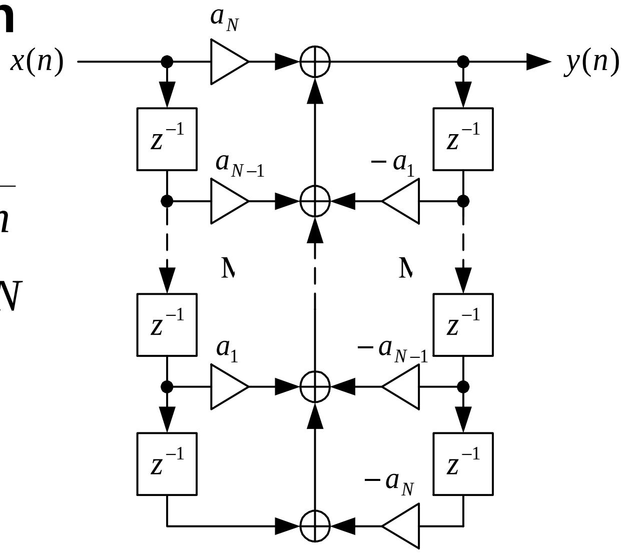
  where D is delay and N is the filter order
- Linear interpolation is obtained with $N = 1$:
  $h(0) = 1 - D$, $h(1) = D$
- Good approximation at low frequencies only

2. IIR FD approximations

- Allpass filters are well suited to FD approximations, since their magnitude response is exactly flat
- The easiest choice is the **Thiran allpass filter** (Fettweis, 1972):

$$a_k = (-1)^k \binom{N}{k} \prod_{n=0}^{N} \frac{D - N + n}{D - N + k + n}$$

$$\text{for } n = 0, 1, 2, \ldots, N$$



- Close relative to Lagrange:

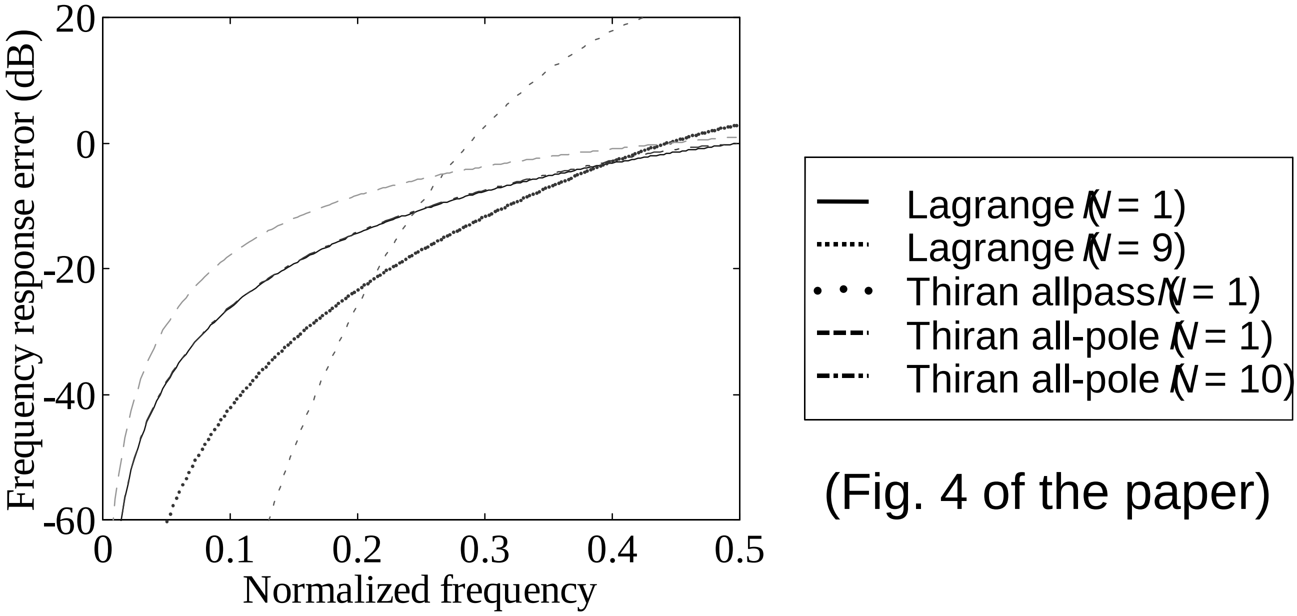
  Max. flat approximation at 0 Hz

3. FD Filters for Very Small Delays

- Very small delays required, e.g., in feedback loops and control applications
  - We consider the case of $D < 1$
- There is always inherent delay in good-quality FD filters
  - Total delay about $N/2$ for FIR and about N for allpass filters
  - Allpass FD filters are stable only for $D > N - 1$
- Thiran all-pole filter (Thiran, 1971) provides small delay
  - Lowpass-type magnitude response cannot be controlled
- FIR filters can approximate small delays but the quality gets low

3. FD Filters for Very Small Delays (2)

- Comparison of various FD filters for a delay $D = 0.5$



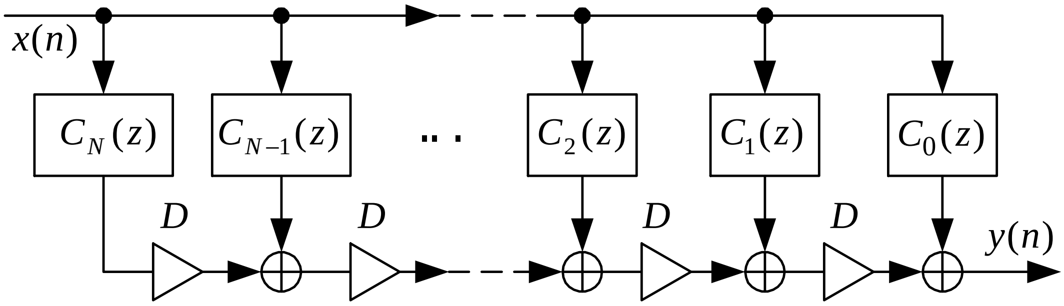
(Fig. 4 of the paper)

4. Time-Varying FD Filters

- Many applications need tunable FD filters
- Three principles to change the coefficients:
  1) Recomputing of coefficients
  2) Table lookup
  3) Polynomial approximation of coefficients
     – Farrow structure (Farrow, 1988)
- FIR filters better suited to TV filtering than IIR filters
  – Time-varying recursive filters suffer from **transients** (we proposed a solution at *ICASSP'98*; see also *IEEE Trans. SP*, Dec. 1998)

4. Time-Varying FD Filters (2)

- Farrow (1988) structure for FIR FD filters
  - Direct control of filter properties by delay parameter *D*



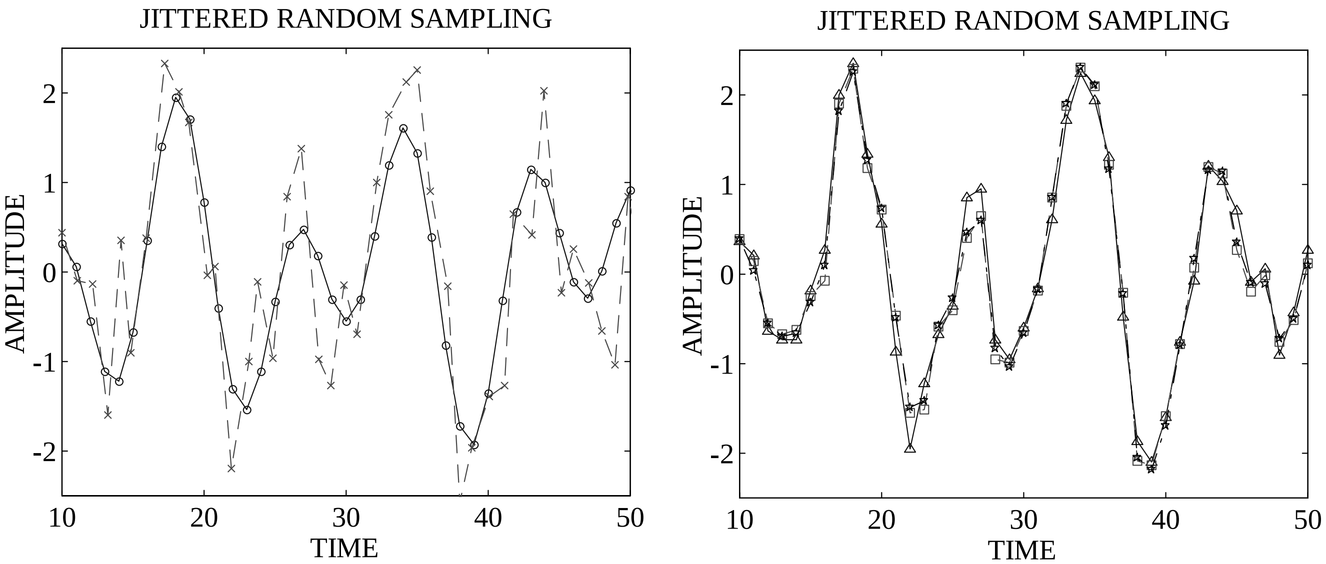
- **Polynomial interpolation filters** can be directly implemented
- Vesma and Saramäki (1996) have proposed a modified Farrow structure and general methods to design the filters $C_k(z)$

5. Polynomial Resampling of Nonuniformly Sampled Signals

- When sampling is nonuniform and sampling instants are known accurately, uniform resampling is possible
  - Problem: traditional sinc series LS fitting computationally intensive and numerically problematic
- Alternative: **polynomial signal model** for smooth (low-frequency) signals
  - Extension of nonuniform Lagrange interpolation
  - Suppress noise also instead of exact reconstruction
  - See: Laakso *et al.*, *Signal Processing*, vol. 80, no. 4, 2000

5. Examples of Nonuniform Reconstruction

- 2 sinusoids plus noise (SNR 3 dB); pol. order 5; filter order 6
- Noise reduction: 3.70 dB (LS reconstruction), 3.43 dB (0th-order appr.) and 3.82 dB (2nd-order appr.)



6. Conclusions

- Fractional delay filters provide a link between uniform and nonuniform sampling
- Useful in numerous signal processing tasks
  - Sampling rate conversion, synchronization of digital modems, time delay estimation, music synthesis, ...
- Resampling of nonuniformly sampled signals on a uniform grid
- MATLAB tools for FD filter design available at: http://www.acoustics.hut.fi/software