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Nonlinear Modeling and Synthesis of the Kantele—a Traditional Finnish String Instrument

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Abstract

We apply a recently developed nonlinear string synthesis algorithm to modeling and synthesis of the kantele, which is a plucked string instrument used in traditional Finnish music for the past few thousand years. The new model combines formerly proposed linear plucked-string models with recent nonlinear extensions. As a result, a dual-polarization nonlinear model is obtained where the difference in the effective string length in two polarizations is accounted for, together with effects caused by the yielding termination of kantele strings and tension modulation. Calibration of model parameters is discussed. The model is also applicable to physical modeling of other plucked string instruments. Sound examples are available at www.acoustics.hut.fi/~vpv/publications/icmc99.htm.

1. Introduction

This paper discusses the design and application of a nonlinear string synthesis model. We combine the formerly proposed linear models [1]–[3] and nonlinear plucked-string models [4]–[6], and propose a new extended structure. It is applied to the physical modeling of a peculiar Finnish string instrument, the kantele [7], [2]. In an earlier work, kantele tones were synthesized with a linear string model that had an instantaneous nonlinearity at its output [7].

The bright and pleasing tone of the kantele is a welcome amendment to the repertoire of instruments in computer music compositions. The synthesis model also enables varying the timbre of kantele or modifying it so that new but physically behaving sounds are generated. The proposed model is applicable to the nonlinear synthesis of other string instruments as well.

2. Acoustics of the Kantele

The kantele has had an important role in traditional folk music in Finland for about 2000 years. The instrument belongs to the class of zithers. It has 5 to 40 strings which are attached in a unique way to a wooden body. The kantele has an interesting, bright tone quality that is characterized by three special features:

1) strong beating due to amplitude modulation of harmonic components,
2) bright timbre caused by strong, slowly decaying harmonics—especially a strong second harmonic, which is still present when a string is plucked close to its midpoint, and
3) audible lowering of the pitch of the tone shortly after the attack—especially in fortissimo playing.

In previous studies, it has been found that the cause for *beats* is the loose knot by which one end of kantele strings is attached to a metal bar [7]. Thus, there is a difference in the effective string length in the vertical and horizontal polarization planes, which brings about the beating.

The strong second harmonic is created by a nonlinear mechanism because the other end of every string is attached (without a bridge) to a tuning peg which is not absolutely rigid [7].

The *descent of the pitch* at the beginning of kantele tones can be heard best in the case of fortissimo playing. This suggests that the phenomenon is caused by tension modulation, which is a direct consequence of the decaying transversal vibration of the string that

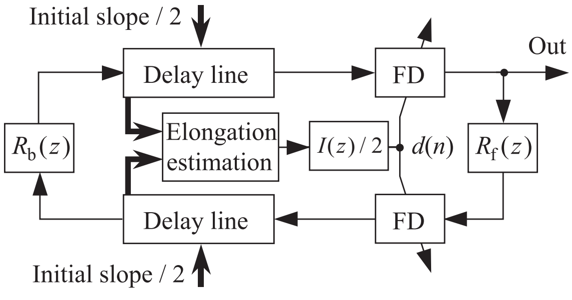

Fig. 1. Waveguide string model where elongation caused by string vibration affects the loop delay through modulation of the fractional delay (FD) filters [5], [6]. The input signal is the initial slope that is inserted into both delay lines.

modulates the transversal wave velocity along the string [8]. Tension modulation also affects the level of harmonic components, thereby modifying the sound in a nonlinear fashion.

In this work, our aim has been to develop a string synthesis algorithm that would faithfully reproduce all the phenomena mentioned above. The model is described in the following.

3. String Model

The suggested new model is based on the principles of digital waveguide modeling [9]. We use a recently developed string model which accounts for the effects of tension variation caused by finite string displacement, i.e., *tension modulation*.

3.1 Nonlinear String Model for One Polarization

Figure 1 shows the block diagram of the tension modulation string model proposed in [5] and [6]. Filters $R_b(z)$ and $R_f(z)$ bring about the frequency-dependent reflection of waves from the end points of the string, as in linear string models [2], and $I(z)$ is a leaky integrator that converts instantaneous elongation estimate into deviation of the delay parameter [5, 6]. Tension modulation is realized using a signal-dependent fractional delay (FD) filter [5], [6], which we implement using a Lagrange interpolation filter [10], [11]. The structure of Fig. 1 can simulate transversal string vibration in one polarization of the string (vertical or horizontal).

3.2 Nonlinear Dual-Polarization String Model

The model structure for each kantele string consists of two parallel waveguide string models, one for each

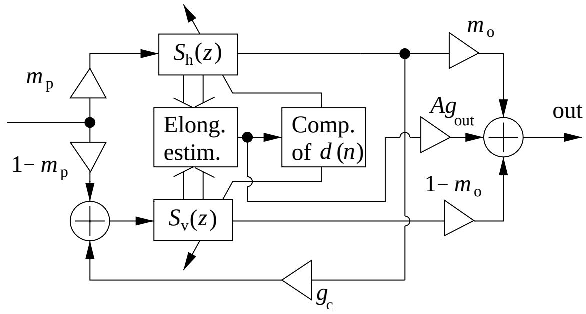

Fig. 2. Block diagram of the dual-mode string model which accounts for the variation of string tension.

polarization, as depicted in Fig. 2, where $S_h(z)$ and $S_v(z)$ are string models identical with that of Fig. 1. As suggested in previous works, the delay line lengths and loop filter parameters of the two models are slightly different to generate beats in the synthetic tone [1]–[3]. The two polarizations may also have different body model filters. Notice that the output signal of the horizontal string model leaks to the input of the vertical one, as suggested in [3], so that a revolving polarization is obtained but the system remains stable irrespective of the value of the coupling coefficient g_c.

The total tension of the string, which controls the fractional delay filters in the model of Fig. 2, is approximated by accounting for signals propagating in both waveguide models. In the dual-polarization vibration, the elongation of the string is given by

$$\lambda_{\mathrm{dev}} = \int_0^{\lambda_{\mathrm{nom}}} \sqrt{1+\left(\frac{\partial y_{\mathrm{h}}}{\partial x}\right)^2+\left(\frac{\partial y_{\mathrm{v}}}{\partial x}\right)^2}\,dx-\lambda_{\mathrm{nom}} \qquad (1)$$

where x is the direction along the string, y_h and y_v are the directions of the horizontal and vertical polarizations, respectively, and λ_{nom} is the nominal string length. In the digital waveguide formulation, Eq. (1) is expressed as

$$L_{\mathrm{dev}}(n) = \sum_{k=0}^{\hat{L}_{\mathrm{nom}}-1} \sqrt{1+p_{\mathrm{h}}^2(n,k)+p_{\mathrm{v}}^2(n,k)} - \hat{L}_{\mathrm{nom}} \qquad (2)$$

where $\hat{L}_{nom}$ is the integer-valued length of the waveguide, $p_h(n, k) = s_{r,h}(n, k) + s_{l,h}(n, k)$ and $p_v(n, k) = s_{r,v}(n, k) + s_{l,v}(n, k)$. The subscripts are r, l, h, and v denote right, left, horizontal, and vertical, respectively. Thus, for example, $s_{r,h}$ is the *right* traveling slope wave in the *horizontal* polarization in the digital waveguide. In practice, we may use the truncated Taylor series approximation of Eq. (2), which yields a simplified formula

$$L_{\text{dev}}(n) = \frac{1}{2} \sum_{k=0}^{\hat{L}_{\text{nom}}-1} p_{\text{h}}^2(n,k) + p_{\text{v}}^2(n,k) \qquad (3)$$

For a more detailed discussion of Eq. (2) and its computationally efficient approximations, see [5].

After the elongation is estimated using Eq. (2) or (3), the time-varying delay parameter $d(n)$ can be computed as [5]

$$d(n) = -\frac{1}{2} \sum_{k=n-1-\hat{L}_{\text{nom}}}^{n-1} (1+A) \frac{L_{\text{dev}}(k)}{L_{\text{nom}}} \qquad (4)$$

where A is the modulation depth parameter related to the string properties and the nominal tension. The summation in Eq. (4) can be replaced with a computationally more efficient leaky integrator $I(z)$, which is included in Fig. 1 [5].

The direct signal path in Fig. 2 from the elongation estimation block to the output (multiplied by Ag_{str}) implements the tension modulation driving force effect, which is described in a companion paper [12].

4. Analysis and Synthesis Examples

The parameters of the nonlinear string model may be calibrated using formerly proposed techniques for the linear part of the system [2], plus a new technique to extract the delay line lengths of the two polarizations, and another novel method to estimate the appropriate tension modulation parameter A [5]. The use of the latter two methods is described below.

4.1 Estimation of the Detuning of Polarizations

The difference in the length of the delay lines of the two models can be estimated based on the beating of kantele tones. We analyzed the envelope of the first harmonic of recorded kantele tones, and extracted the beating frequency by fitting a sine wave using the least squares method. Figure 3 presents an example where the linear trend and mean of the envelope of the first harmonic have been removed, and a sine wave has been fitted to the data. We use an average of two cases, a horizontal and a vertical pluck. This method yields a good estimate of the difference in the fundamental frequency of vibration of the two polarizations.

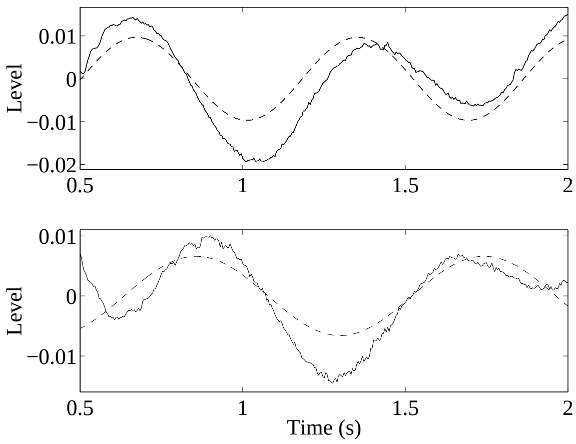

Fig. 3. Beating of the first harmonic (solid line) in a kantele tone plucked horizontally (top) and vertically (bottom), with a least squares sine wave fit (dashed line).

Table 1. Fundamental frequencies (in Hz) of the two string models of Fig. 1 for kantele synthesis, and their difference.

String	#1	#2	#3	#4	#5
$f_{0,\text{h}}$	466.5	415.6	392.4	355.5	315.1
$f_{0,\text{v}}$	465.2	414.5	391.3	354.5	314.3
Δf_0	1.3	1.0	1.1	0.97	0.77

Table 1 shows the fundamental frequencies corresponding to the two delay lines of the synthesis model in Fig. 1 for the five strings of the kantele. The f_0 of the horizontal string model has been chosen to be the nominal value based on pitch analysis of recorded kantele tones, and that of the vertical string model has been set Δf_0 smaller. Generally speaking, the difference Δf_0 between the fundamental frequencies is around 1 Hz (see Table 1), which can also be heard in recorded kantele tones.

4.2 Estimation of Tension Modulation Depth

The tension modulation parameter A can be estimated from the pitch variation of a recorded tone [5]. Since the deviation depends on the amplitude of the tone—in both recorded and synthetic tones—we have to choose the numerical value of the input amplitude to be used in synthesis prior to estimating A.

An example of variation of the fundamental frequency is given in Figure 4. The maximum fundamental frequency here is $f_{0,\max}$ = 327.7 Hz, and the

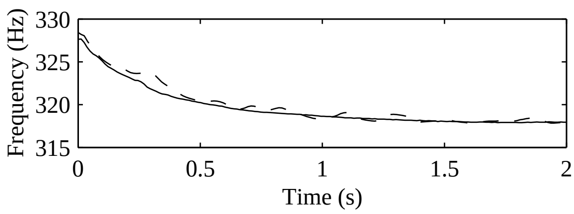

Fig. 4. Variation of the fundamental frequency in a kantele tone as result of a fortissimo pluck (solid line) and a synthetic tone (dashed line).

nominal value is $f_{0,nom}$ = 317.7 Hz. When the sampling rate is 22.05 kHz, the overall f_0 variation, 10.0 Hz, corresponds to a change of –1.06 samples in the delay lines of the synthesis model of Fig. 1. Assuming that the displacement is 4.0 mm, the value of *A* will be 1315 (for details in computing *A*, see [5]).

We now present an example of resynthesizing the kantele tone produced with a fortissimo pluck. The values of *A* and maximum displacement are those mentioned above. The loop filter coefficients are g = 0.9975 and a_1 = –0.0200 for both polarizations, and $f_{0,max}$ = 317.7 Hz. In the dual-model model, m_p = 0.5000, g_c = 0.0010, and m_o = 0.8000. The string was plucked at the midpoint, although this was not the case of the recorded tone. In Fig. 4, the variation of the fundamental frequency of this synthetic tone is presented with a dashed line. A good match with the analyzed result can be observed.

5. Conclusions and Future Plans

This paper has described a nonlinear string model that can be used in the simulations of plucked string instruments. In this paper, the application to the synthesis of the kantele was studied. The algorithm is, however, a general model for a vibrating string that exhibits tension modulation. The special characteristics of the kantele, such as beating and observable descent of the tones, are also present in synthetic tones. The tension modulation nonlinearity causes coupling of harmonic modes of the string, as described elsewhere [5], [6]. Examples of recorded and synthetic kantele sounds are available at www.acoustics.hut.fi/~vpv/publications/icmc99.htm. In the future, we will employ this method in physical modeling of the tanbur, a traditional Turkish string instrument [13].

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