MODEL BASED CURVE FITTING FOR IN-SITU SURFACE IMPEDANCE MEASUREMENT

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Abstract: This paper describes model-based curve fitting for surface impedance measurements. The method is used to make in-situ measurement results more robust and reduce the resolution problems at low frequencies due to unavoidable time-domain windowing of reflection responses. When the physical structure of the material to be measured is known, an acoustic model having few parameters can be generated. This model can be used as a reflectance transfer function to match the impedance and the acoustic behavior of the surface. With this method many kind of surfaces can be modeled, e.g. layered absorbents, air gaps, and different combinations of these. The models are derived from the physical properties of materials and wave propagation theory, and they can be fitted to measurement results by nonlinear optimization (curve fitting). This model fitting approach can also be used as a general methodology to parameterize the behavior of different surface structures.

Keywords: in-situ measurement, material modeling

NOTATION

R	reflection coefficient
R_f	flow resistivity
Z_0	characteristic impedance
g	propagation coefficient
k	wave number
r	density
a	absorption coefficient

1. INTRODUCTION

In-situ impedance measurements have been studied for some time and there has been numerous propositions for different techniques. Compared to the laboratory methods, the in-situ measurements are strongly affected by the surrounding space. The parasitic reflections from surrounding surfaces needs to windowed out and this sets a limit for low-frequency resolution.

Many of the proposed methods use the plane wave approximation which is only valid when the source distance is much greater than the wave length ($r >> \lambda$) and the angle of incidence is small [1,2]. Especially at low frequencies the far-field requirement is hard to fulfil and this decreases accuracy when the plane wave approximation is used.

In general the in-situ impedance measurement methods can be divided into two main groups: transient and stationary.

In the first group the impedance is calculated by comparing the transient responses of the direct and the reflected sound. The impulses can be separated by windowing but this decreases the frequency resolution even more. In the subtraction method [3], there is no need for windowing the direct and the reflected sound from the impulse response. This improves the low frequency resolution considerably.

In stationary methods the direct and the reflected sound are left to combine and the impedance is calculated from the resulting interference pattern. The impedance can be calculated by comparing the measurement to some reference (free-field or hard surface) measurement [4,5] or as an inverse boundary problem. Free-field two-microphone transfer function methods [6,7] and free-field methods with one microphone [8] are examples of the latter.

Quite often the physical structure of the material to be measured is known and this information can be used to increase the accuracy and the reliability of the measurement. An acoustic model having few parameters can be generated and by nonlinear optimization the reflectance transfer function from the model can be fitted to the measurement data.

On the other hand, the same technique can be used when creating models of already existing surfaces. A low-order function can be fitted to a measurement data and the resulting model can be used, e.g., to parameterize different kinds of materials and surface structures.

In this paper we will introduce the curve fitting technique and apply it for empirical sound propagation models introduced by Delany and Bazley [9] and Mechel [10]. Also a low order generic system model is generated and fitted to simple case, where a homogenous absorbent is placed on a rigid wall.

2. MODELS FOR CURVE FITTING

There are many kinds of empirical and semi-empirical, as well as purely theoretical models for acoustical wave propagation in material. The major drawback in most of the models is that there are too many parameters for curve fitting. As a result of extensive laboratory measurements, in 1970 Delany and Bazley [9] published empirical equations for the propagation coefficient γ and the characteristic impedance Z_0 in fibrous absorbents. The only parameter to measure was the flow resistivity R_f. Later, there has been many variations of the equations to improve the accuracy, especially at low frequencies, and validity for different materials [10,11]. In this paper, only the equations published by Delany and Bazley are presented, but naturally other models can be used the same way.

3. IMPEDANCE AND REFLECTION COEFFICIENT

For simplicity and to introduce the technique, the waves are assumed plane. The spherical wave laws and oblique incidence could be taken into account in model-based curve fitting as well. The incident and the reflected pressure wave are noted as

$$p_i = (x) = p_0 e^{-gx} \quad \text{and} \quad p_r(x) = R p_0 e^{gx} \qquad (1,2)$$

where R is the reflection coefficient and $g=jk$ is a complex propagation constant. The corresponding particle velocities can be written as

$$u_i(x) = \frac{p_i(x)}{Z_1} \quad \text{and} \quad u_r(x) = -\frac{p_r(x)}{Z_1}, \qquad (3,4)$$

where Z_1 is the characteristic impedance of the medium. The impedance seen by the total pressure field is now

$$Z(x) = \frac{p_i(x) + p_r(x)}{u_i(x) + u_r(x)} = Z_1 \frac{e^{-gx} + \mathrm{R}e^{gx}}{e^{-gx} - \mathrm{R}e^{gx}}. \qquad (5)$$

If the medium is air ($Z_1=r_0c$) and the surface is at $x=0$ the surface impedance can be solved

$$z(0) = \frac{1+R}{1-R}. \qquad (6)$$

Here the impedance $z(0)$ is normalized to the characteristic impedance of air $Z_0=r_0c$. Correspondingly the reflection coefficient can written

$$R = \frac{z(0)-1}{z(0)+1}. \qquad (7)$$

Either the reflection coefficient R or the surface impedance Z is obtained from the in-situ measurement. The absorption coefficient can be calculated when either one of these is known. The absorption coefficient is defined as

$$a(f) = 1 - \left|R(f)\right|^2. \qquad (8)$$

In general the normal reflection coefficient for plane waves is defined as

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1}. \qquad (9)$$

By substituting Eq. (9) in to Eq. (5) and after rearranging the terms, the impedance can be written as

$$Z(x) = Z_1 \frac{Z_2 + Z_1 \tanh(gx)}{Z_1 + Z_2 \tanh(gx)}. \qquad (10)$$

With this equation the impedance can be transferred anywhere when the impedances Z_1, Z_2 and the propagation constant g is known.

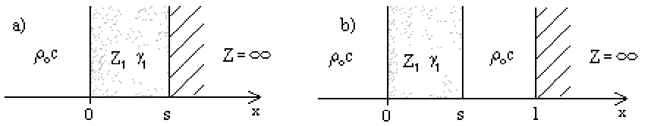

Figure 1. a) An absorbent on a hard wall and b) an absorbent and a hard wall with an air gap.

4. MODELS

4.1. Acoustic wave propagation models

Fig. 1a shows an absorbent on a rigid wall and Fig. 1b shows the same absorbent on a rigid wall with an air gap. If the impedance and the propagation coefficient of the material were known, Eq.(10) could be used to estimate the impedance at the surface.

According to Delany and Bazley the flow resistivity g and the characteristic impedance Z_c can be estimated with the following equations:

$$g = \left(j \frac{w}{c} \right) \left[1 + 0.0978 \cdot B^{-0.700} - j \cdot 0.189 \cdot B^{-0.595} \right] \qquad (11)$$

$$Z_c = r_0 c \left[1 + 0.0571 \cdot B^{-0.754} - j \cdot 0.087 \cdot B^{-0.732} \right] \qquad (12)$$

where B is a dimensionless frequency-dependent variable:

$$B = \frac{r_0 f}{R_f} \qquad (13)$$

The r_0 the density of the medium, f is the frequency (in Hertz), R_f is the flow resistivity, and c is the speed of sound.

If an absorbent is placed on a rigid wall (Fig. 1a) and the impedance of the wall is assumed $Z_2 = \infty$, the equation (10) can be written in the form of

$$Z(x) = Z_1 \coth(gx), \qquad (14)$$

where Z_1 is the characteristic impedance of the absorbent and g is the propagation constant. These values are obtained from the Delany-Bazley equations (11) and (12). When the thickness x of the absorbent is known, the impedance can be solved at the front surface of the material. The only parameter left to solve is the flow resistance R_f. This can be used as a free parameter in curve fitting. The reflection coefficient, calculated from this model is fitted to the measured data.

The same method and model can easily be used with air gaps too. Figure 1b shows a case with an air gap between a hard wall and an absorbent. With Eq. (10) the impedance is first mapped from the surface of the wall over the air gap to the back of the absorbent ($Z_c = r_0 c$), and then from the back of the absorbent to the front of the surface. The characteristic impedance and the propagation constant are obtained from the Delany-Bazley equations.

4.2. Non-physical models

If the general behavior of the material is known, any model that behaves the same way can be used as a system model for the surface. As in Fig. 2, the surface under study can be considered to have a certain transfer function with a set of free parameters.

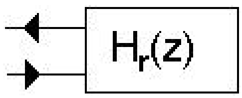

Figure 2. A case where the reflection from the surface is modeled with a transfer function $H_r(z)$.

As an example, a low-order model is generated for a homogenous wool-like absorbent on a hard wall. In general, such a system behaves very much like a low-pass filter. Thus, the system could be modeled by a first-order zero-pole filter $H_r(z)$.

$$H_r(z) = \frac{k}{1 + az^{-1}} + b, \qquad (15)$$

where k is a constant, b sets the high frequency gain and a sets the cut-off frequency. The absorbent is assumed to have no loss at zero frequency ($\alpha=0$). Thus, by setting $H_r(1)=1$, k can be solved

$$k = 1 + a - b - ab. \qquad (16)$$

Now k can be substituted to Eq. (15) and the model can be written as

$$H_r(z) = \frac{1 + a - ab - abz^{-1}}{1 + az^{-1}}. \qquad (17)$$

The high-frequency gain b and the cutoff frequency parameter a are left as free parameters.

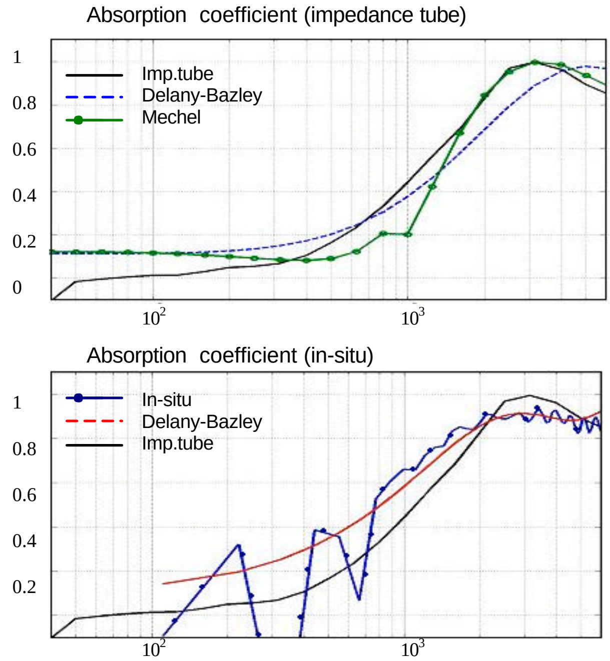

Figure 3. Fitting to stone wool (18 mm, 60 kg/m^3). In the lower figure the Delany-Bazley model is fitted to in-situ measurement data. In the upper figure the Delany-Bazley and the Mechel model is fitted to impedance tube data.

4.3. The fitting the model

First, by using Eq. (10) the impedance can be mapped from the surface of the hard wall to the surface of the absorbent. The impedance of the wall can be set to have a very high value or if the wall impedance is assumed infinite then Eq. (14) can be used instead. The propagation coefficient g and the characteristic impedance Z_1 of the absorbent are obtained from the Delany-Bazley equations (11) and (12). Now the impedance at the surface of the absorbent is a function of flow resistivity R_F. By substituting this impedance to Eq (9), a reflectance function can be written, which is also a function of flow resistivity R_F:

$$R(R_f) = \frac{Z - r_0 c}{Z + r_0 c}, \qquad (18)$$

where Z is the estimated surface impedance and $r_0 c$ is the characteristic impedance of air.

Next this reflectance function is fitted to the measured reflection coefficient by using the flow resistivity R_F as a free parameter. As a result a value for the flow resistivity is obtained. Now by using this flow resistivity value the Eq. (18) is a complete model of the surface under study.

For the low-pass model Eq. (17) was directly fitted to the measured reflection coefficient. The cutoff frequency parameter a was used as a free parameter and the high frequency gain b was set to a fixed value.

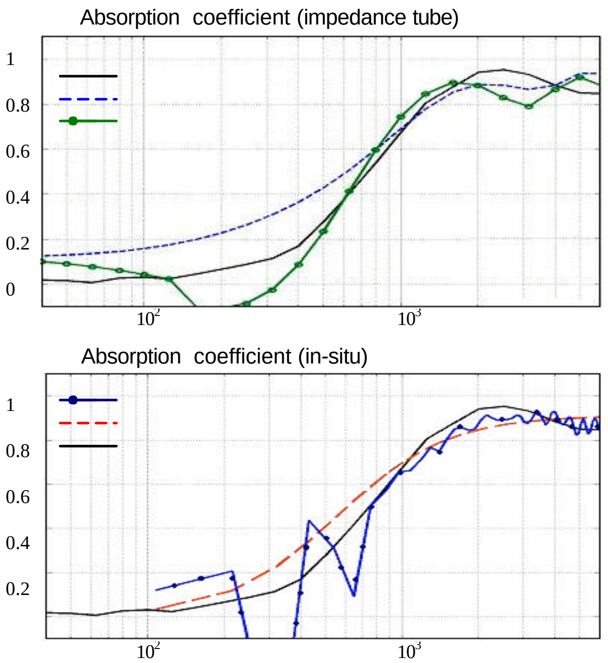

Figure 4. Fitting to glass wool (50 mm, 40 kg/m^3). In the lower figure the Delany-Bazley model is fitted to in-situ measurement data. In the upper figure the Delany-Bazley and the Mechel model is fitted to impedance tube data.

5. Results

The model-based curve fitting was tested with in-situ impedance measurement data and for comparison the results were compared with impedance tube measurements. All the computing was done with MATLAB and for curve-fitting a least-square-based non-linear optimization function

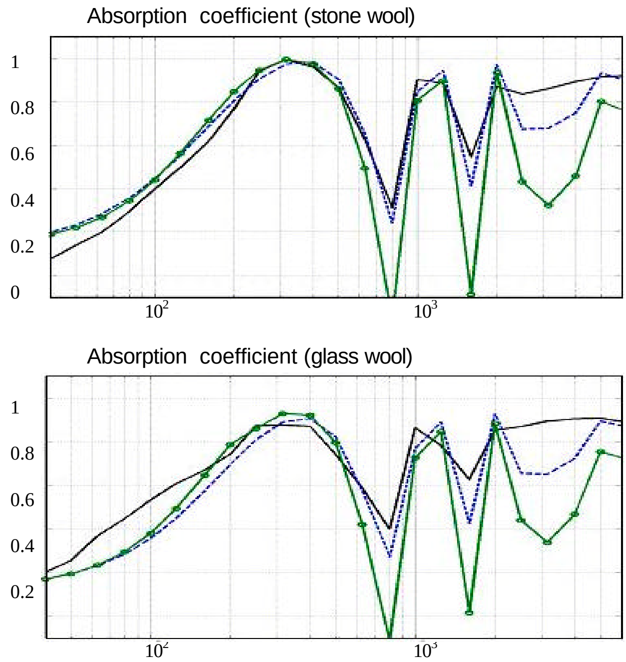

Figure 5. The Delany-Bazley (dashed) and the Mechel (-o--)models fitted to glass wool (50 mm, 40 kg/m^3) and stone wool (18 mm, 60 kg/m^3) impedance tube data (solid) with a 200 mm air gap.

lsqcurvefit( ) was used. The in-situ measurements were done by using the subtraction method [3]. The sample sizes were 120cm x 120cm. The absorbent was placed on a concrete floor.

To also test the robustness of the method, fairly poor quality in-situ measurements were chosen. Naturally, the better the measurement data, the more reliable results fitting gives. To simulate a very accurate in-situ measurement the fitting was also applied to impedance tube measurements. In addition to Delany-Bazley model also the Mechel [10] model was fitted to impedance tube data. Mechel made some modifications to Delany-Bazley equations to increase the accuracy at low frequencies. There are many modifications to the equations by Delany and Bazley and naturally any of those could be used.

In Fig. 3 the models are fitted to stone wool (18 mm, 60 kg/m^3) measurement data and Fig. 4 shows the fitting done to glass wool (50 mm, 40 kg/m^3) measurement data. Even though the in-situ measurement data has a lot of fluctuation the model fitting can still give fairly good results. The fitting to the impedance tube data shows that with better quality in-situ measurements the fitting would give even more accurate result, especially at low frequencies.

The Delany-Bazley and the Mechel model were also fitted to impedance tube measurement data with a 200 mm air gap. Fig. 5 shows the results with the same materials as in the previous case. Both models seems to give good results up to

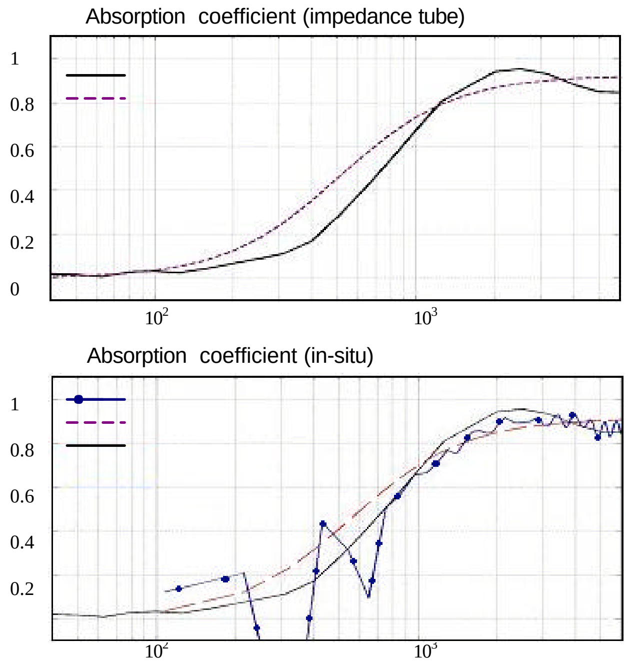

Figure 6. Fitting to glass wool (50 mm, 40 kg/m^3) measurement data. In the upper figure the fitting is done to impedance tube data and in the lower figure the fitting is done to in-situ measurement data.

2000 Hz. This was expected because the models are not intended to be used at high frequencies.

The non-physical low-pass filter model for homogenous wool like absorbents was also tested. In Fig. 6 the filter model is fitted to glass wool (50 mm, 40 kg/m^3) measurement data. For this case the model seems to fit very well and especially at low frequencies the model follows the impedance measurement data extremely well. It was noted during the testing that for homogenous materials the filter models were surprisingly accurate.

6. Conclusions

In this paper a model-based curve-fitting method for in-situ surface impedance measurements was introduced. According to the measurements the method can give fairly reliable results and it can be used for increasing the reliability and robustness of in-situ measurements, especially at low frequencies. Naturally the method can be used in other measurements as well. Also a non-physical model was tested and for simple cases this is found useful.

7. Acknowledgement

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8. REFERENCES

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