

A Digital Bandpass/Bandstop Complementary Equalization Filter with Independent Tuning Characteristics

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Abstract—A discrete-time realization of first-order (shelving) and second-order equalization filters is developed, providing bandpass/bandstop magnitude-complementary transfer functions. The bandpass transfer function is turned into a complementary one, and vice versa, switching between two structures that share a common allpass section containing the state variables of the equalizer. The present realization is an alternative to existing solutions, particularly in applications where the equalization parameters are dynamically varied.

Index Terms—Audio systems, equalizers, magnitude-complementary transfer functions, tunable filters.

I. INTRODUCTION

LINEAR equalization is a well-known processing technique, used when a musical signal needs improvements in “presence” or exhibits artifacts that can be corrected through a manipulation of its spectrum. In the digital domain, graphic equalization can be efficiently performed using first-order and second-order *tunable equalization filters* [1]. In such structures, the low-frequency (LF) and high-frequency (HF) gains (first-order equalization filters) and the center-frequency gain (second-order equalization filters) are determined by the value of one multiplying coefficient. Other tuning parameters, i.e., the cutoff frequency of first-order filters, and the selectivity and center frequency of second-order filters, are embedded in the coefficients of an allpass block, whose freedom of realization is another valuable feature of these structures [2].

Although such filters are minimum phase, their gain responses cannot be complemented (more specifically, their transfer functions cannot be reciprocated) just by varying the gain coefficient. Many solutions have been proposed to overcome this problem. In some cases, proper functions have been designed, mapping the gain parameter over the filter coefficients of the equalizer [3], [4]. In other cases, the allpass section of the tunable equalization filter has been preserved during the calculation of the inverse transfer function, yielding an efficient realization of the complementary equalizer [5].

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In this letter, a different approach is presented. Given a minimum-phase transfer function $H(z) = 1 + A(z)$, where $A(z)$ is an allpass filter where pure unit delays cannot be factored out, the inverse $1/H(z)$ can be realized using a procedure that preserves most of the structural properties of the original structure, in particular the allpass block [6].

With this procedure, we can compute the inverse transfer function of a bandstop tunable equalization filter. In this way, we obtain a complementary bandpass response avoiding the design of a new, independent filter. The complete bandpass/bandstop equalization filter is made of two different structures that are alternatively used anytime the response switches from bandpass to bandstop and vice versa. In spite of this, the allpass section is left untouched by such commutation.

Compared with the Regalia–Mitra filter [1], a real-time implementation of the proposed system demands more computation cycles on a digital signal processor. However, the lookup of filter coefficients is simplified. Its performance becomes valuable in application cases involving dynamic variations of the gain parameter.

II. SYNTHESIS OF THE BANDPASS TRANSFER FUNCTION

The transfer function of a bandstop tunable digital equalizer H_{BS} can be put in the form

$$H_{BS}(z) = \frac{1+K}{2} \left\{ 1 + \frac{1-K}{1+K} A(z) \right\}, \quad 0 < K < 1 \quad (1)$$

where $A(z)$ is an allpass filter [1]. When $A(z)$ is a first-order allpass then we have an LF shelving filter. When $A(z)$ is a second-order allpass, then we have a second-order equalizer. An LF shelving filter is turned into an HF by changing the sign of $1-K$ in (1). The value of K determines the edge-frequency cut in the case of shelving filters, and the notch-frequency level in the case of second-order equalizers. In both cases, H_{BS} is minimum-phase.

Let $H_{BP}(z)$ be the inverse of $H_{BS}(z)$. An inversion of the bandstop transfer function aiming at preserving the allpass block induces the *delay-free loop problem* [7, Sec. 6.1.3]. In fact, as long as a new value of the input signal x is acquired at time-step n , we cannot explicitly calculate the bandpass filter output y making use of the output l from the allpass

$$y[n] = \frac{2}{1+K} x[n] - \frac{1-K}{1+K} l[n] \quad (2)$$

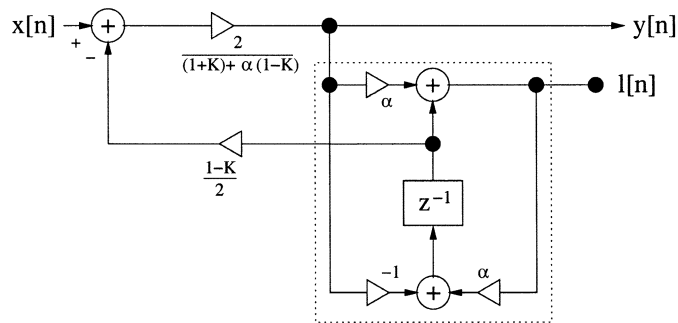


Fig. 1. Switch-free structure for the computation of the bandpass transfer function of a shelving filter. The allpass section, realized in transposed canonic form, is located inside the rectangle in the dashed line. The terms $1 - K$ must be changed into $K - 1$ when performing HF shelving.

since the computation of $l[n]$ requires the knowledge of $y[n]$.

Nevertheless, we can look at $l[n]$ as a linear combination of two components, i.e., $l[n] = \alpha y[n] + l_0[n]$: the first component, accounting for the delay-free part of the allpass response, contains the coefficient α that sets the cutoff frequency and the selectivity of first- and second-order equalizers, respectively [1]; l_0 is the output from the allpass filter fed with zero. By substituting this formula in (2), it can be seen that $y[n]$ can be computed at each time-step using the following procedure [6]:

- 1) $l_0[n]$ is computed feeding the allpass filter with zero.
- 2) $y[n]$ is calculated by

$$y[n] = \frac{2}{(1+K) + \alpha(1-K)} \left\{ x[n] - \frac{1-K}{2} l_0[n] \right\}. \quad (3)$$

- 3) The allpass filter is fed with $y[n]$ to update its state variables.

This procedure can be computed providing the filter structure with switches that, during each time-step, alternatively feed the allpass block with zero or $y[n]$, respectively implementing Steps 1) and 3) of the procedure. The use of switches allows to design the allpass independently of the rest of the structure. Otherwise, switches can be avoided if we reconsider the allpass output decomposition as $l_0[n] = l[n] - \alpha y[n]$ [8]. As an example, Fig. 1 shows a switch-free structure that embeds an allpass filter in transposed canonic form, realizing a shelving filter. This example can be immediately extended to second-order realizations, substituting the first-order allpass with a second-order allpass in the same form.

A structure that includes switches for computing the above procedure is included as part of the system depicted in Fig. 2, inside the rectangle in the dashed line. In that structure, a hold block (labeled with H) retains the output that is calculated when the switches are in position I [Steps 1) and 2) of the procedure], then feeds the allpass block when the switches are in position II [Step 3)].

The whole discussion still holds if the range of K is changed into $1 < K < \infty$, i.e., if the roles of H_{BS} and H_{BP} are swapped. Such a choice is supported by the definition of cutoff frequency and filter selectivity [1] governing the value of α . However, the former assumption results in filter coefficients whose range better accommodates fixed-point number representations.

III. IMPLEMENTATION OF THE EQUALIZER

A system realizing both the bandpass and bandstop transfer function is given in Fig. 2. When the switches are in the position labeled with BS, the system implements a bandstop equalization filter. When they switch to the position labeled with BP, the system becomes a complementary (bandpass) filter. In principle, transitions between the two structures do not introduce transients in the output signal and the internal state. In fact, they happen when $K = 1$: under this condition, the position of the switches is insignificant.

Fig. 3 shows (in the dashed line) plots of six gain responses from the original Regalia–Mitra LF shelving filter for gains equal to -12 , -8 , -4 , 4 , 8 , and 12 dB, respectively, together with plots (solid line) of the responses from the proposed system having the same gains of the Regalia–Mitra filter when working in bandstop configuration. In all cases, $\alpha = 0.8668$. Fig. 4 shows similar gain responses for the second-order equalizer (dashed line), centered at a normalized frequency equal to 0.01, together with gain responses from the proposed system (solid line), complementing the bandstop Regalia–Mitra equalizer ($\alpha = 0.9844$). Moreover, six responses coming from a second-order equalizer that has been designed following the Bristow–Johnson approach [3] are also plotted, for the same gains and setting the center frequency to 0.21.

In the proposed system, additional computations are needed to calculate $l_0[n]$ and $y[n]$. In particular, they reduce to one sum plus one multiply to compute $y[n]$ and one multiply followed by one sum to compute $l_0[n]$, if the allpass section is realized with a lattice structure. One table lookup is required to load the value $G_{BP} = 1/\{(1+K) + \alpha(1-K)\}$ each time a variation in the parameter K occurs. A multiplication by two must be applied to the input signal when the switches are set to position BP. Conversely, the output signal is divided by two when the system is in bandstop configuration.

Such result seems valuable if we compare the system in Fig. 2 with Regalia–Mitra filters, in front of variations of the gain parameter. With those filters, we can require magnitude-complementary bandpass/bandstop transfer functions. If we choose to preserve the independence of gain tuning in the bandstop filter, then a complementary bandpass response can be obtained by tuning the parameter α_{BP} , which will be used in place of α in bandpass configuration, according to the value of $K > 1$

$$\alpha_{BP} = \frac{1 - K^{-1} + \alpha(1 + K^{-1})}{1 + K^{-1} + \alpha(1 - K^{-1})} \quad (4)$$

(as usual, when performing HF shelving we have to change $1 - K^{-1}$ into $K^{-1} - 1$).

Hence, a complementary tunable equalization filter requires looking up one table stored with coefficients $K < 1$ when providing the bandstop response, and two tables stored with values $K > 1$ and α_{BP} , respectively, when providing the bandpass transfer function (such values are properly scaled when they are represented in fixed-point arithmetic).

The proposed system equals a tunable equalization filter when providing a bandstop response. Since the bandpass transfer function is obtained through a switch, it needs neither a table stored with $K > 1$ nor a table accounting for α_{BP} .

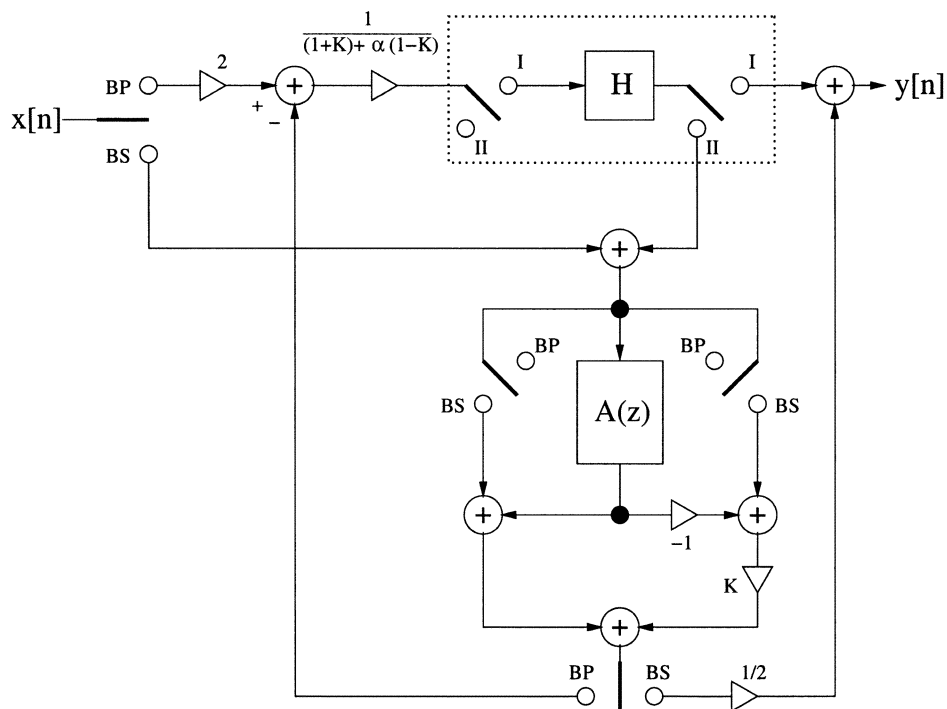


Fig. 2. Bandpass/bandstop equalizer. When switches are set to position BS, a bandstop transfer function is provided. When switches are set to position BP, a complementary (bandpass) transfer function is provided. A switching structure (located inside the rectangle in the dashed line) containing a hold block H is employed for calculating the inverse transfer function. HF shelving is realized by swapping the branch containing the multiplier by K with the one terminating at the adder common to both of them.

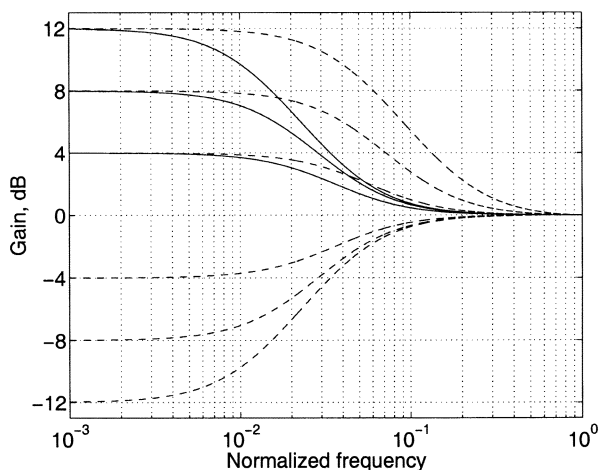


Fig. 3. Gain responses from the Regalia-Mitra LF shelving filter for gains equal to -12 , -8 , -4 , 4 , 8 , and 12 dB (dashed line). Gain responses from the proposed system for gains equal to -12 , -8 , -4 dB (solid line). In both cases $\alpha = 0.8668$.

However, it needs a table stored with G_{BP} that is looked up according to the value $K < 1$. For this reason, the coefficients in the system have magnitudes that are inherently smaller than unity.

In summary, a comparison between the proposed system and complementary Regalia-Mitra equalizers, made with respect to variations of the gain parameter, shows that the former solution prevents from storing gain coefficients $K > 1$, needed for bandpass filtering. This advantage is paid computationally, since two more sums and two more multiplies must be executed during each time-step in bandpass configuration.

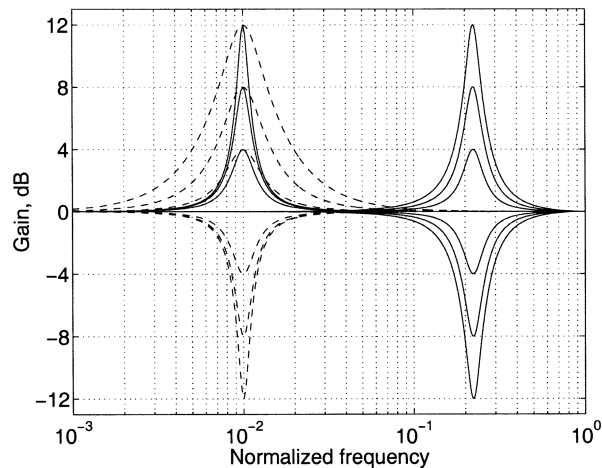


Fig. 4. Gain responses from the Regalia-Mitra second-order equalizer for gains equal to -12 , -8 , -4 , 4 , 8 , and 12 dB (left, dashed line). Gain responses from the proposed system for gains equal to -12 , -8 , -4 dB (left, solid line). In both cases, $\alpha = 0.9844$. Normalized center frequency set to 0.01 . Gain responses from the Bristow-Johnson equalizer for gains equal to -12 , -8 , -4 , 4 , 8 , and 12 dB (right, solid line). Normalized center frequency set to 0.21 .

IV. SUMMARY

A digital equalizer has been presented. It integrates previously known tunable equalization filters that are used here to provide the bandstop transfer function. The system switches to a different structure providing a complementary (bandpass) transfer function; meanwhile, it preserves the internal state and most of the structural properties of those filters. This property results in less memory consumption and simplified mapping of the equalization parameters over the filter coefficients.

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