MODELING AND REAL-TIME SYNTHESIS OF THE KANTELE USING DISTRIBUTED TENSION MODULATION

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ABSTRACT

Nonlinear behavior of a vibrating string is responsible for acoustical features in some plucked-string instruments, resulting in a characteristic and easily recognizable tone. That is also the case for the Finnish kantele, a traditional plucked-string instrument used in folk music. Earlier works have analyzed the general acoustic properties of the kantele and discussed related sound synthesis techniques. In this study, a novel modeling and sound synthesis method for simulating nonlinear string vibrations with spatially distributed tension modulation is presented. The modeling is conducted through a Digital Waveguide (DWG) approach, using controllable fractional delay elements in implementing the distributed tension modulation nonlinearity. The elongation of the vibrating string is estimated and the result is used in tuning the fractional delay values accordingly. Because of the spatially distributed nature of the approach, control of the string model parameters and observation of its behavior can be implemented at any point along it, in contrast to prior digital waveguide string models. This new approach is applied in constructing a physical model of a five-string kantele. Real-time sound synthesis is implemented using an efficient, block-based modeling tool, the BlockCompiler.

1. INTRODUCTION

During the previous years physical modeling has become a more and more tempting sound synthesis approach when the objective is to synthesize the sound of a musical instrument. Possible growth of computational complexity when physical models are used is well compensated by increased sound quality and physically meaningful control data. This study uses a technique called Digital Waveguide modeling (DWG) in establishing a computational model for sound synthesis of nonlinear vibrating strings, in order to produce a physically based sound synthesis of the kantele. The DWG method is widely used in sound synthesis, and it is especially well suited for modeling vibrating strings. An excellent introduction to DWG's used in modeling musical instruments can be found in [1].

2. ACOUSTICAL ANALYSIS OF PLUCKED STRINGS

An ideal plucked string with rigid terminations at both ends will produce a harmonic sound, where the frequency of every partial will be an integer multiple of the fundamental frequency. If the string behaves linearly (i.e., a change in the string length does not change the behavior of the vibrational movement), the fundamental frequency will be constant throughout the vibration process. It is well known also that an ideal nonelastic vibrating string can be plucked in such a way that some of the harmonics are missing. More specifically, if the string is plucked at position 1/N of its length, the resulting motion will lack every Nth harmonic.

The behavior of a nonlinear string is however somewhat different. In an elastic string, the plucking (and vibration) introduces an increase in the length of the string, which results in a growth of tension and furthermore an increase in the frequency of the vibrational movement. As the vibration decays, the effect of the nonlinearity is diminished, and the frequency of the vibration approaches asymptotically the frequency in the linear case.

Also, the behavior of missing harmonics is different in a nonlinear string if the string terminations are nonrigid. The tension modulation in the string will cause excitation of the missing harmonics via the nonrigid string ends. In a way, the energy of other modes will “leak” to the missing modes through the mode coupling in the termination points. The missing harmonics will therefore experience a gradual onset after the pluck, and after that they will decay exponentially like all other harmonics. The generation of missing modes in nonlinear vibrating strings is covered in more detail by Legge and Fletcher [2].

3. PLUCKED-STRING MODELS USING DIGITAL WAVEGUIDES

The DWG string model relies on the d’Alembert’s solution of the one-dimensional wave equation to divide the wave into two components: the left-proceeding and the right-proceeding wave. The behavior of the string can be seen as the superposition of these two waves. In the DWG method, the string is seen consisting of two delay lines, where the signals travel in opposite directions and proceed to the other delay line when the end of the first one is reached. If the model represents a vibrating string, the reflection of wave components at the endpoints must include a sign change.

Resistive losses of the vibrational movement are modeled with attenuating coefficients somewhere along the delay lines. Frequency-dependent losses in the vibrating string are often modeled with a lowpass filter, called the loop filter, usually implemented at the termination point of the delay line [3], [4]. Since most of the sound radiated from plucked stringed instruments is coming from the vibrating body or the top plate of the instrument, it is logical to consider the output signal as a force signal to the point connecting the body and the string. In the case of a DWG model this would correspond to taking the output as the velocity difference of the two delay lines at the end of the waveguide.

4. ACOUSTICAL ANALYSIS OF THE KANTELE

The kantele is a bridgeless, traditional Finnish plucked string instrument, with usually five metal strings in its basic form. The strings are terminated at one end by metal tuning pins, which are screwed directly into the soundboard. At the opposite end all strings are wound once around a horizontal metal bar called the varras and then knotted. Because of the nonzero distance between the center of the varras and the knot, the vibrations in two polarization planes will have different effective lengths; the varras will be the termination point for horizontal vibration, while the knot will act as a termination for the vertical vibration. This phenomenon causes the total vibration of the string to have two fundamental components with slightly different frequencies, producing beating. The structure of the kantele and the string termination at varras is illustrated in Figure 1. A more detailed structure and acoustical analysis of the kantele can be found in an earlier work [5]. An acoustically improved new design and a study of the history of kantele is presented elsewhere in these proceedings [6].

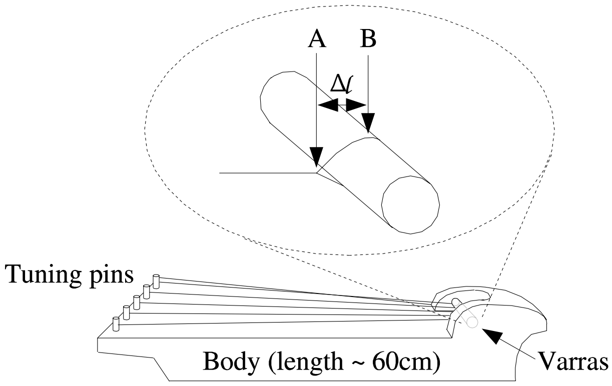

Figure 1: Illustration of the kantele. The string termination at varras is magnified for clarity. A denotes the termination point for vertical vibration of the string, while B denotes the termination point for horizontal vibration. $\triangle l$ stands for the distance between A and B.

5. NOVEL KANTELE STRING MODEL

5.1. Implementing the effects of tension modulation

From the discussion in Section 2, a conclusion can be drawn that simulation of the effects of tension modulation is important if high-quality plucked-string instrument synthesis is to be obtained. Tension modulation in a one-dimensional digital waveguide model is best modeled using fractional delay elements (also known as fractional delay filters). The fractional delay elements are basically allpass filters with adjustable coefficients, being able to produce a controllable delay [7], [8]. They are widely used in the literature with one-dimensional DWG's in tuning the delay lines [3],[9]. The design process of the fractional delay elements is covered in [8].

Previous works [4], [10], [5] use a single fractional delay element in a single-polarization string model for computational efficiency. Physically this would correspond to a single elastic element at the termination point of an otherwise rigid string. An alternative solution can be obtained if the elongation process can be distributed along the delay line, in a similar way that a real physical string functions, where the elasticity is distributed along the string.

Elongation of the string can be expressed as [2]

$$l_{dev} = \int_0^{l_{nom}} \sqrt{1 + (\frac{\partial y}{\partial x})^2} dx - l_{nom} \quad (1)$$

where l_{nom} is the nominal string length, x is the spatial coordinate along the string, and y is the displacement of the string. The first spatial derivative in equation (1) would suggest using slope as a wave variable in the DWG, and thus equation (1) can be approximated for the digital waveguide as [10]

$$L_{dev}(n) = \sum_{k=0}^{\hat{L}_{nom}} \sqrt{1 + [s_r(n,k) + s_l(n,k)]^2} - \hat{L}_{nom} \quad (2)$$

where $s_r(n,k)$ and $s_l(n,k)$ are the slope waves at position k and time instant n, propagating to the right and to the left, respectively. $\hat{L}_{nom}$ is the rounded nominal string length. To reduce the computational complexity, equation (2) can be further simplified using a truncated Taylor series expansion to [10]

$$L_{dev}(n) \approx \frac{1}{2} \sum_{k=0}^{\hat{L}_{nom}} [s_r(n,k) + s_l(n,k)]^2 \quad (3)$$

while still maintaining a sufficient accuracy. The approximated delay variation of the total DWG can be obtained from equation (3) as [10]

$$D_{tot}(n) \approx -\frac{1}{2} \sum_{l=n-1-\hat{L}_{nom}}^{n-1} (1 + \frac{ES}{F_{nom}}) \frac{L_{dev}(l)}{L_{nom}} \quad (4)$$

where E is Young's modulus, S is the cross-sectional area of the string, and F_{nom} is the nominal tension corresponding to the string at rest.

Since the system under consideration uses a distributed set of delay elements, the total delay must be divided by the number of delay elements in the DWG to obtain a delay value for each individual element. The fractional delay element used in the system is a first-order allpass filter of the form

$$A(z) = \frac{-a + z^{-1}}{1 - az^{-1}} \quad (5)$$

where a is the filter coefficient which defines the length of the delay. Notice that when $a = 0$ the allpass filter acts as a unit delay. Coefficient a can be expressed as [7]

$$a = \frac{1 - d_{partial}}{1 + d_{partial}} \quad (6)$$

where $d_{partial}$ is the delay intended for a single allpass filter. The first order allpass-filter is the best choice for the fractional delay element when delay values around unity are to be obtained [11]. Generally, the approximation of an allpass filter is most accurate when the desired delay is close to the order of the allpass filter. First-order allpass filters also have relatively low computational complexity and zero magnitude response error. The phase response error caused by the allpass filter is not considered to pose a problem since its effect is negligible in the audio frequency range, assuming the sampling frequency to be reasonably high [7].

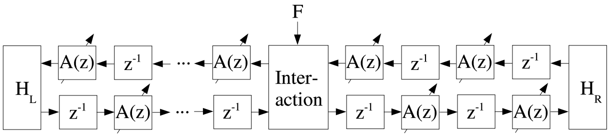

Figure 2: The single-polarization string model. $A(z)$ denotes an allpass filter, while z^{-1} denotes a unit delay. H_L and H_R are FIR filters modeling the frequency-dependent losses at the left and right string termination points, respectively. The interaction element provides means for inserting a force signal into the string, and its location can be changed.

5.2. Single polarization string model

A single-polarization string model constructed from the elements discussed above is illustrated in Figure 2. In this model, both delay lines carry velocity waves, thus allowing a velocity signal of any location on the string to be obtained by summing the left- and right-proceeding wave components at that particular location. Slope waves can be obtained by taking the difference of the wave components and scaling it by a constant coefficient. The unit delays between each allpass filter ensure that no delay-free loops will emerge even if the position of the interaction element is changed.

It is possible to implement an alternative string model, where all delay lines are constructed from allpass filters, resulting in a more homogeneous structure. With this approach however, it must be assured that the structure has at least one unit-delay element inside each loop in order to avoid computability problems. A more in-depth study concerning the construction of the single-polarization model is presented by the authors [12].

5.3. Dual polarization string model

The novel synthesis model of a single kantele string is constructed using two single-polarization string models connected through a coupling impedance. The elongation approximations of the strings and the resulting allpass filter coefficient values are evaluated separately for the two string models. The structure of the novel kantele string model is illustrated in Figure 3. In this model, $v_1(n)$ and $v_2(n)$ represent the velocity signals coming from the strings vibrating vertically and horizontally, respectively.

It is important to note that while $v_1(n)$ and $v_2(n)$ can be obtained anywhere along the string, in this case they are evaluated at the termination points, so that terminal impedances can be used. $Z_{1bridge}$ and $Z_{2bridge}$ represent the vertical and horizontal terminal impedances, respectively. Z_c stands for the coupling impedance from vertical to horizontal string vibration polarization, and $f_{12}(n)$ represents the corresponding driving force. The forces to the termination caused by the two one-polarizational vibrations are denoted by $f_1(n)$ and $f_2(n)$. The output of the whole two-polarization string model is presented by force signal $f_{out}(n)$ excerted to the string termination point.

6. SYNTHESIS RESULTS

The synthesis results reveal that the behavior of the nonlinear string model is consistent with the theory of elastic strings, discussed in Section 2. The fundamental frequency behaviors of a single-polarization string model and a real kantele tone are compared in

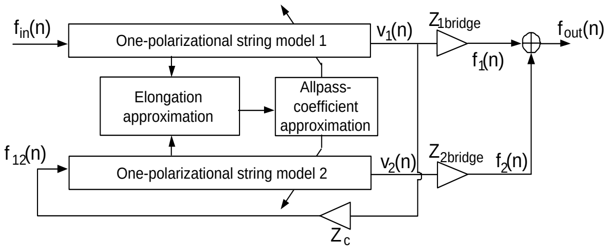

Figure 3: Two-polarization kantele string model. The one-polarization string model blocks are identical to what is illustrated in Figure 2. Both one-polarization blocks have different lengths. It is important to note that the coupling between the vibrational polarizations in a real physical system is more complicated but a simplified one-way coupling is used here for clarity.

Figure 4. In this figure, the dash-dotted line approximates the mean value of perceptual detection threshold of an initial pitch glide. The detection threshold in the frequency region of these tones is approximated to be 5.4 Hz [13]. The string model used in this figure had a total delay line length of 55.25 and the allpass coefficient a was scaled using a constant value of 0.8 in order to simulate the behavior of the recorded sample. The sampling frequency was 44.1 kHz for all string models used in this paper.

The envelope trajectories of the first three partials of a nonlinear plucked string model tone are depicted in Figure 5(a), while the same partials obtained from a linear plucked string model are illustrated in Figure 5(b). As predicted in Section 2, the missing harmonic in the nonlinear case has a gradual increase after the onset of the vibration, and after that it experiences an exponential decay like all other modes. The presence of the heavily attenuated third harmonic in the linear case in Figure 5(b) can be explained by the fact that the excitation to the string was not positioned exactly but only approximately at the location 1/3 of the string's length. The linear string model used in Figure 5(b) was the same as the nonlinear one except that the allpass-coefficient was kept constant during the whole process. The string model used in Fig. 5 had total delay line lengths of 60.5 samples including two third-order lowpass filters (with coefficients $[-0.0761\ 0.566\ 0.566\ -0.0761]$) at both ends as loop filters, and the interaction element was placed between the 20th and 21st delay element. The beating discussed in Section 4 is naturally absent from Fig. 5, since only a single-polarization string was modeled.

A real-time sound synthesis model of a kantele is constructed using an experimental, block-based, efficient audio-DSP-tool, the BlockCompiler. The algorithm used is efficient enough to process a kantele model with several strings on an ordinary laptop computer at a 44.1 kHz sampling rate. A more detailed description of the BlockCompiler is presented by Karjalainen [14].

7. CONCLUSIONS

In this paper we have presented a spatially distributed model of a nonlinear vibrating string. The acoustic features of a real elastic string (fundamental frequency drift and generation of missing harmonics) are modeled in a physically correct manner with this novel approach. The string model is constructed using digital waveguides, and fractional delay filters are used for simulating the nonlinearities. A kantele synthesis model is constructed using two

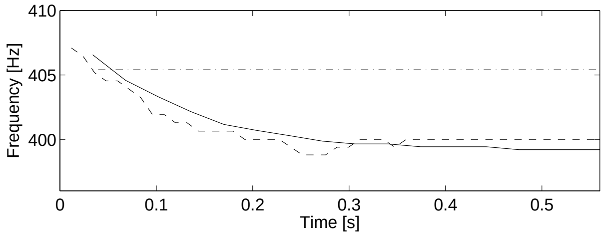

Figure 4: Illustration of the fundamental frequency glide in the synthesized tone (solid line) and in a real kantele tone, obtained via measurements (dashed line). The approximated detection threshold of a pitch drift, illustrated as a dash-dotted line, suggests that the fundamental frequency drifts in both cases are audible.

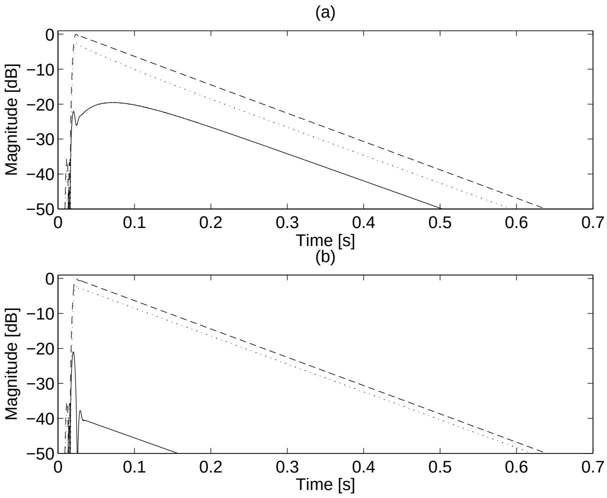

Figure 5: Envelope trajectories of first three harmonics of a synthetic tone in the nonlinear case (a) and in the linear case (b). The string was plucked in both cases at approximately 1/3 of its length. The first harmonic is plotted with a dashed line, the second with a dotted line, and the third with a solid line.

coupled string models for each real kantele string. The advantage of the new approach over previous methods for synthesizing nonlinear strings is its spatially distributed structure, which allows the interaction to the string model to be applied at any point along it.

8. ACKNOWLEDGMENTS

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