DSP Formulation of a Finite Difference Method for Room Acoustics Simulation

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ABSTRACT

The digital waveguide mesh is a finite difference scheme for simulation of wave propagation in acoustic spaces. In this paper we show how this scheme can be formulated in terms of digital signal processing. A transfer function for a node in an N-dimensional mesh is derived. The main emphasis is in three-dimensional systems such as simulation of room acoustics. Wave propagation in the mesh is analyzed by numerical simulations and visualizations. Setting boundary conditions to walls is also studied.

1. INTRODUCTION

The method presented in this paper is based on using digital waveguides. The method is called waveguide mesh method. It is a finite difference time domain (FDTD) algorithm. The FDTD technique is widely used in electromagnetics, but also in simulation of room acoustics [1].

One-dimensional digital waveguides are a discrete numerical method widely used to model musical instruments, such as string and wind instruments [2]. Also two-dimensional (2D) and three-dimensional (3D) extensions of waveguides have been used for plates and drums [3], and room acoustics [4], [5].

In this paper we present a digital signal processing formulation for this method.

2. MULTI-DIMENSIONAL WAVEGUIDE MESH

Digital waveguide modeling is based on discretizing time and space. In this study the discretization points are called nodes. Nodes are connected by bi-directional digital waveguides of length of unit delay. The discretization points and their connections form a digital waveguide mesh.

In modeling one-dimensional systems, such as vibrating strings, the waveguide mesh forms one bi-directional delay line. Each node has two neighbors except the termination nodes which have only one neighbor.

For multi-dimensional systems each node may have several neighbors. Their number depends on the chosen mesh topology. Thus each node has several unit delay element connections and it acts as a scatterer between the digital waveguides.

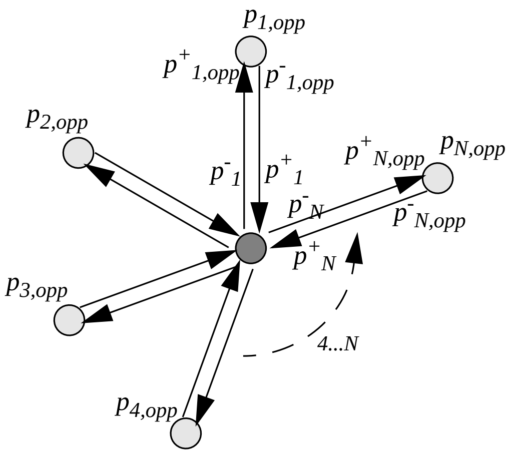

Fig 1. *A scattering junction with N neighbors connected by bi-directional unit delay elements.*

2.1. Transfer Function for a Scattering Junction

In this section a general equation is derived for a node with *N* neighbors. The sound pressure in a waveguide *i* is represented by p_i, volume velocity by v_i, and impedance of the waveguide by Z_i. The volume velocity is equal to pressure divided by impedance ($v_i = p_i/Z_i$). The delay elements are bi-directional and the sound pressure of the waveguide is defined as the sum of its input (p^+) and output (p^-)

$$p_i = p_i^+ + p_i^- \tag{1}$$

Two conditions must be satisfied at a lossless junction connecting *N* bi-directional delay lines as in figure 1:

1. The sum of input velocities equals the sum of output velocities (flows add to zero)
2. The sound pressures in all crossing waveguides are equal at the junction (continuity of impedances).

By using above mentioned conditions sound pressure at a node can be expressed as

$$p = 2\sum_{i=1}^{N}\frac{p_i^+}{Z_i}\Big/\sum_{i=1}^{N}\frac{1}{Z_i} \tag{2}$$

The basic principle with unit-delay waveguides is that the input from a waveguide is the same as output in its opposite end one time step before. This can be expressed as the equation

$$p_i^+ = z^{-1} p_{i,\,opp}^- \tag{3}$$

where $p_{i,\,opp}^-$ presents output at opposite end of the waveguide *i*. By using equations (1) and (3) the sound pressure in a node takes the following form

$$p = \frac{2\sum_{i=1}^{N} \frac{z^{-1} p_{i,\,opp}}{Z_i}}{\sum_{i=1}^{N} \frac{1}{Z_i}} - \frac{2\sum_{i=1}^{N} \frac{z^{-1} p_{i,\,opp}^+}{Z_i}}{\sum_{i=1}^{N} \frac{1}{Z_i}} \tag{4}$$

By applying once again the same equations we get an expression for sound pressure

$$p = z^{-1} \frac{2\sum_{i=1}^{N} \frac{p_{i,\,opp}}{Z_i}}{\sum_{i=1}^{N} \frac{1}{Z_i}} - z^{-2} p \tag{5}$$

Equation (5) can be expressed also in form of a transfer function

$$p = \frac{z^{-1} \cdot 2\sum_{i=1}^{N} \frac{p_{i,\,opp}}{Z_i} \Big/ \sum_{i=1}^{N} \frac{1}{Z_i}}{1 + z^{-2}} \tag{6}$$

In a usual case the medium is homogenous and impedances in all waveguides are equal and equation (6) can be reduced to the form

$$p = \frac{z^{-1} \cdot \frac{2}{N} \sum_{i=1}^{N} p_{i,\,opp}}{1 + z^{-2}} \tag{7}$$

Verbal description of equation (7) is that the sound pressure in a node is two times the average of its neighbors subtracted by its own value two time units ago. This expression is independent of the dimensionality of the system. Only the number of neighbors is relevant.

2.2. Mesh Excitation

Exciting the mesh must be done so that none of the equations (1)-(5) is violated. If we want to create an excitation of value *I* in a homogenous mesh at a node with *N* neighbors, we set the incoming sound pressure to all waveguides connected to it. Using equation (2) the value for sound pressure is found to be

$$p_i^+ = \frac{I}{2} \qquad \forall i = 1 \ldots N \tag{8}$$

To keep the equation (3) satisfied the history of neighbors has to be updated also, since the outgoing sound pressures in opposite ends of waveguides at one time step before excitation must equal to *I/2*. Because there are no other outputs, the sum of output signals is also *I*/2, which

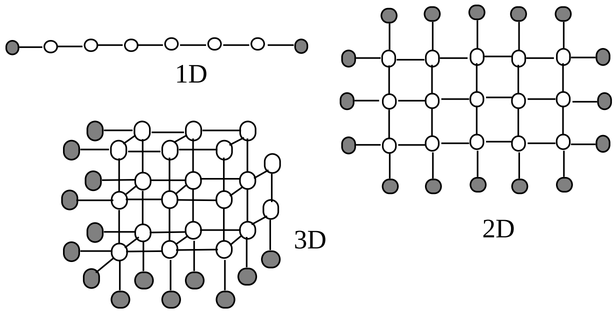

Fig 2. *Example structures of rectilinear 1D, 2D and 3D meshes.*

is the same as the sum of input signals. By using equation (2) again we come to the result that all the neighbor nodes must have value *I*/*N* at one time step before excitation.

3. MESH STRUCTURE

There are different topologies which may be used to construct a digital waveguide mesh. The only restriction is that all the delay-elements connecting nodes must be of equal length. If there are waveguides of different lengths, the equation (5) is no more valid. Instead, equation (4) can be used, z^{-1} is replaced with z_i^{-x}, and *x* represents the delay length of waveguide *i*.

The requirement of equal length waveguides is satisfied by any regular space decomposition method. It means that the atomic tiles of decomposition are polytopes whose sides are of equal length [6]. In this study we concentrate on rectilinear meshes, since they are computationally efficient and easy to construct. Some hexagonal, triangular and tetrahedral structures both in 2- and 3-dimensional systems are presented in [7] and [8].

An N-dimensional rectilinear waveguide mesh is a regular array of digital 1-D waveguides arranged along each perpendicular dimension, interconnected at their crossings. In figure 2 there are examples of meshes of different dimensionalities.

In an *N*-dimensional rectilinear mesh each node inside the mesh has exactly *2*N* neighbors. Nodes on the boundaries have only one neighbor. Due to this arrangement the boundaries can be terminated the same way as in digital 1D waveguides.

3.1. Sampling Frequency of a Rectilinear Mesh

Distance between mesh nodes together with the mesh topology determines the sampling rate of the mesh. In a one-dimensional system waves propagate one unit distance in one time unit since the waveguide mesh is constructed of unit-delays.

In a multi-dimensional rectilinear mesh effective wave propagation speed is found at the diagonal direction. In an *N*-dimensional system one diagonal movement, corresponding to distance of $\sqrt{N}$ unit distances, requires *N* unit delays. Thus the update frequency of a rectilinear *N*-dimensional mesh is

$$f = \frac{c\sqrt{N}}{dx} \tag{9}$$

where c represents the speed of sound in the medium and dx is the unit distance between two nodes. For example in air $c \approx 340 m/s$, and if a 3D mesh is discretized with spacing of 5 cm, the update frequency will be ca. 12 kHz.

3.2. Rectilinear 3D Waveguide Mesh

For studying the behavior of rectilinear 3D waveguide mesh we have made numerical simulations. Equation (7) can be expressed also as a difference scheme

$$p(n) = \frac{1}{3}\sum_i p_{i,\,opp}(n-1) - p(n-2) \tag{10}$$

where $p_{i,opp}$ are the axial 3D neighbors of a node, and n represents time. This scheme is equivalent to the difference equation derived from the wave equation by discretizing time and space with forward-time-center-space method. Simulations are made in a mesh with 3 cm discretization grid, corresponding to sampling frequency of ca. 20 kHz.

In figure 3 the magnitude responses registered at different distances along a 2D diagonal of the mesh are shown. The figure shows that doubling the distance from the sound source attenuates signal 6dB like it should do in a real 3D environment.

The shape of the transfer function is due to the nature of excitation. The source is point like and its impedance is frequency dependent so that low frequencies are attenuated. The magnitude response is symmetrical to half of the Nyquist frequency and for that reason only the first half is shown.

Figure 4 presents two impulse responses in time domain. The first one is registered in axial and the second in 3D diagonal direction. The responses are quite different from each other. The diagonal response has ideal attenuation slope, but the axial response is distorted. The reason for this is that the mesh is anisotropic. Wave propagation characteristics are direction dependent.

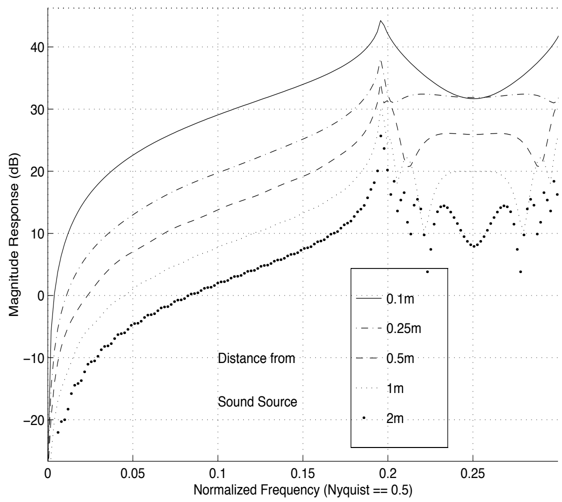

Fig 3. *Magnitude response registered from various distances at diagonal direction.*

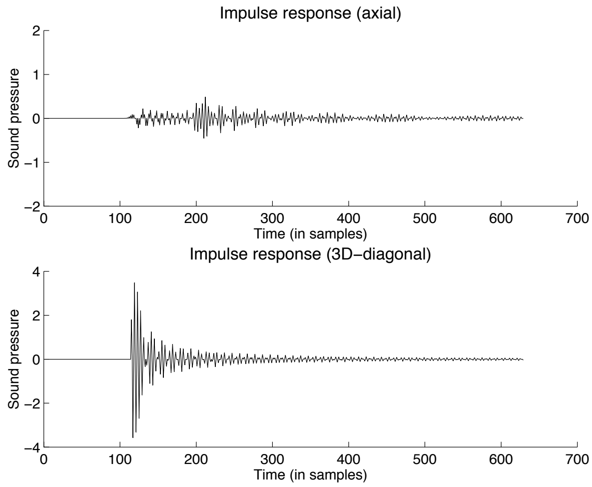

Fig 4. *Impulse responses to axial and 3D-diagonal directions at the same distance from the sound source.*

Figure 5 shows the effect of mesh anisotropy to magnitude response. The difference is largest between axial (az=0˚, el=0˚) and diagonal (az=45˚,el=45˚) propagation. Signal has attenuated in axial direction at higher frequencies. Up to one fifth of the Nyquist frequency the magnitude response to all directions is within 2 dB.

Figure 6 shows the dispersion characteristics of the mesh. In the figure there are group delays to three different directions. Both the 2D and 3D diagonals introduce no obvious dispersion while in axial direction the dispersion is remarkable above one fifth of the Nyquist frequency.

Considering both the magnitude and phase the valid frequency range for simulations is from DC to about one tenth of the sampling frequency.

4. BOUNDARY CONDITIONS

The most straightforward way to set boundary conditions is the mesh structure where each boundary node has only one neighbor. There are also other possible ways, but in this study we concentrate on the 1D termination. If we have a boundary with reflection coefficient r, the transfer function for the boundary node is

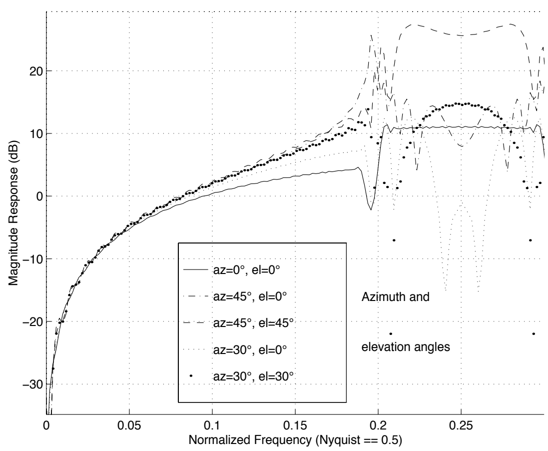

Fig 5. *Magnitude response from constant distance to various angles.*

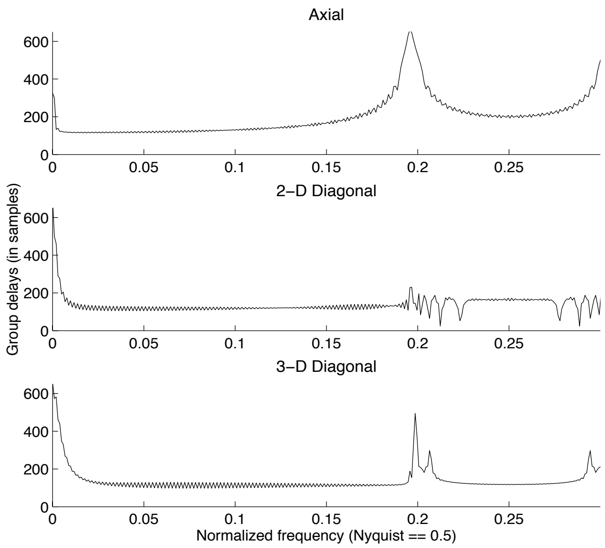

Fig 6. *Group delays in three different directions, axial, 2D diagonal and 3D diagonal.*

$$p = \frac{(1+r)\cdot z^{-1} p_{opp}}{1+rz^{-2}} \qquad (11)$$

This result is found by using equation (4) and setting $p^{+}_{wall} = 0$, since there is no input from inside the wall. In addition the following relation between impedance change and reflection factor is used: $r = \frac{Z_2 - Z_1}{Z_2 + Z_1}$. Equation (11) can also be expressed as a difference equation

$$p(n) = (1+r)\cdot p_{opp}(n-1) - r\cdot p(n-2) \qquad (12)$$

where p is the boundary node and p_{opp} is its neighbor. In general the boundary condition can be replaced by any digital filter.

Figure 7 shows behaviour of some simple boundary conditions. The model consists of three rooms constructed of different materials. The first figure shows the room configuration, excitation shape and first order phase preserving reflection from the ceiling. In the second one there are phase reversing reflections from the walls. The walls separating rooms are anechoic.

5. SUMMARY AND FUTURE WORK

In this paper we have shown that finite difference time domain method can also be formulated in terms of digital signal processing. The study is based on digital waveguides.

In the future we are going to make this algorithm to better suit real room acoustic problems. This includes a detailed study on boundary conditions and also on coupling of different sound propagation mediums. To enhance dispersion characteristics various interpolation techniques are under study, and some results are described in a companion paper [9].

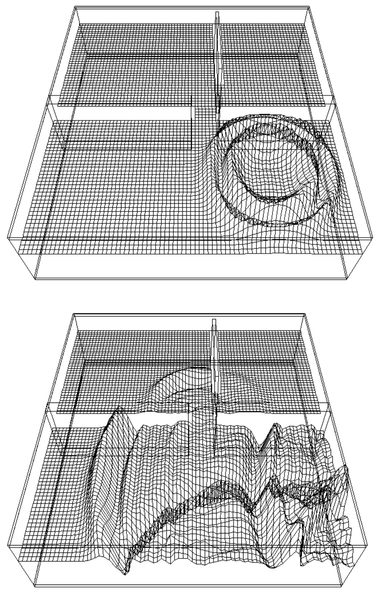

Fig 7. *Visualization of simulation, excitation and first order reflection (above), wall reflections (below).*

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