
An Overview of New Techniques and Effects in Model-based Sound Synthesis

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Abstract

Physical modeling and model-based sound synthesis have recently been among the most active topics of computer music and audio research. In the modeling approach one typically tries to simulate and duplicate the most prominent sound generation properties of the acoustic musical instrument under study. If desired, the models developed may then be modified in order to create sounds that are not common or even possible from physically realizable instruments. In addition to physically related principles it is possible to combine physical models with other synthesis and signal processing methods to realize hybrid modeling techniques. This article gives an overview of some recent results in model-based sound synthesis and related signal processing techniques. The focus is on modeling and synthesizing plucked string sounds, although the techniques may find much more widespread application. First, as a background, an advanced linear model of the acoustic guitar is discussed along with model control principles. Then the methodology to include inherent nonlinearities and time-varying features is introduced. Examples of nonlinearities are studied in the context of two string instruments, the kantele and the tanbur, which exhibit interesting nonlinear effects.

1 Introduction

Physical modeling and model-based sound synthesis have been studied with increasing activity twenty-thirty years (Hiller & Ruiz, 1971; Jaffe & Smith, 1983; Smith, 1983; Smith, 1996). For sound synthesis purposes the aim of modeling is to find signal processing algorithms that imple-

ment the physically and perceptually most important properties of a specific instrument or of a family of instruments, enabling real-time synthesis with good fidelity. The approach has been most successful in modeling of one-dimensional acoustic resonators that can be considered as linear and time-invariant (LTI) systems. Vibrating strings and air columns are good examples thereof (Fletcher & Rossing, 1991). Two-dimensional acoustic systems, such as membranes and plates, as well as three-dimensional resonators, such as rooms, have also been modeled successfully, although computational complexity grows rapidly with dimensionality.

Memoryless nonlinearities have been added to model-based synthesis, too. For example reed functioning in wind instruments can be realized this way fairly accurately (Smith, 1987; Välimäki et al., 1992; Cook, 1992). Also the interaction between piano hammer and string has been simulated with models where the nonlinearity happens in a single point and is computed as parameter changes at specific time moments only (Van Duyne et al., 1994). More general forms of nonlinearity and signal-dependent time variance, i.e., non-LTI modeling, have not yet progressed very far. There are several reasons to this: (a) nonlinear models tend to be specific cases and a general methodology cannot be as systematically formulated as for LTI systems, (b) building nonlinear models and estimating their parameters is inherently more difficult than for LTI systems, and (c) real-time computation of nonlinearities requires usually more processing power than LTI models. Due to rapid progress in processor capacity, however, the non-LTI approach is becoming increasingly attractive since it allows for improved reality and nuances in model-based sound synthesis.

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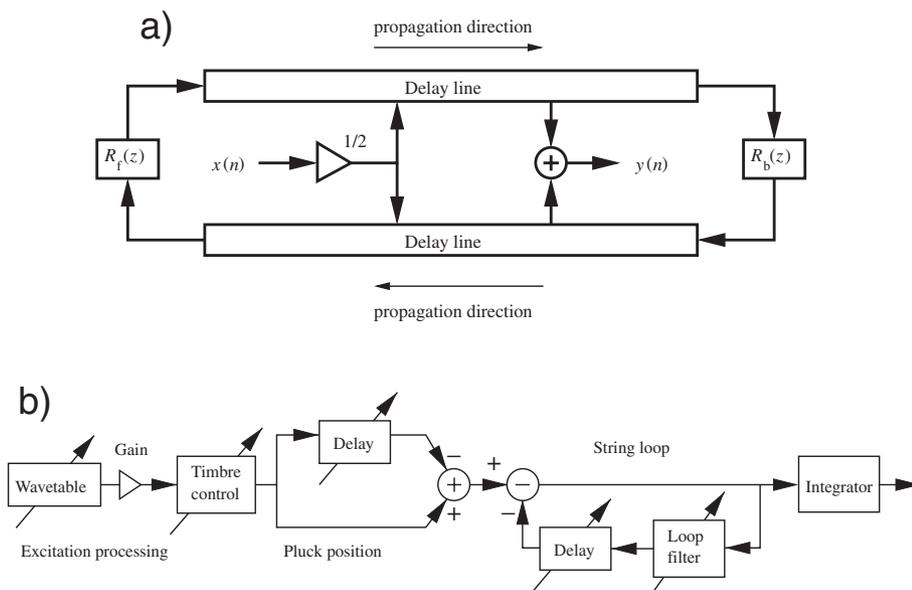


Fig. 1. Reduction of a dual delay line digital waveguide model (a) into a single delay loop model (b). Filters $R_f(z)$ and $R_b(z)$ model wave reflection at string terminations. $x(n)$ is pluck excitation and $y(n)$ pickup output (bridge output in the acoustic guitar). The single delay loop model (b) consist of wavetable excitation, timbre and pluck position filters, string loop with delay and attenuation (loop filter) controls, and an integrator.

This overview article concentrates mostly on a specific family of instruments, the plucked string instruments, by discussing some recent developments. It starts from advanced LTI synthesis models and their control methods and focuses then on nonlinearities and signal-dependent time-variant features. Examples are given in the context of two inherently nonlinear string instruments, the Finnish kantele and the Turkish tanbur.

2 Linear string models

In 1983 Karplus and Strong (Karplus & Strong, 1983) published a simple algorithm that created quite realistic string and percussive sounds. As shown by Jaffe and Smith (Jaffe & Smith, 1983) and Smith (Smith, 1983), it was a rudimentary physically related model needing extensions for more accurate modeling of physical reality. Smith developed the ideas of physical modeling and model-based synthesis further, resulting in the digital waveguide synthesis techniques (Smith, 1987; Smith, 1992; Smith, 1997), an important principle for discrete-time modeling of acoustic systems. Since then this approach has been developed to cover a wide range of musical instruments that exhibit one- or two-dimensional resonators (Van Duyne & Smith, 1993). The principle has been successfully extended also to three-dimensional systems, such as rooms and loudspeaker enclosures (Savioja et al., 1996).

The one-dimensional digital waveguide model is very well suited to the synthesis of plucked string instruments (Karjalainen & Laine, 1991; Välimäki et al., 1996). Smith (1983) formulated the fundamental properties of such

models. The two-directional digital waveguide model of Figure 1a has separate delay lines for the left and right traveling wave components. Input $x(n)$ is injected to an excitation point and output $y(n)$ is taken from a pickup point (the bridge in an acoustic guitar). The string terminations reflect most part of the traveling waves back to the string and these reflections can be modeled by filters $R_f(z)$ and $R_b(z)$.

A detailed derivation of reduced string models was presented by Karjalainen et al. in (Karjalainen et al., 1998). It was shown how the two-directional digital waveguide model can be reduced to a more efficient *single delay loop* (SDL) model, shown in Figure 1b. It is computationally very efficient, yet complete for modeling LTI string vibration in one polarization, as far as proper parameter values of the model and a plucking excitation are given (Tolonen, 1998). The SDL loop filter includes all effects to the wave traveling from one end of the string to another and back. The loop filter can be divided into parts: (a) attenuation control, (b) delay control, and (c) optional allpass filter – not shown in Figure 1b – for dispersive wave propagation in a stiff string. A special technique for good approximation of the exact delay – corresponding to desired pitch of a sound – is called fractional delay filtering. Fractional delay filtering techniques and their applications are discussed in (Laakso et al., 1996; Välimäki, 1995).

An important principle for efficient synthesis was proposed in (Smith, 1993) and (Karjalainen et al., 1993b), the commuted body modeling consolidated with plucking excitation. The body of a string instrument can be considered as an LTI transfer function that transmits string vibration at the bridge to radiated sound field. This includes all body reso-

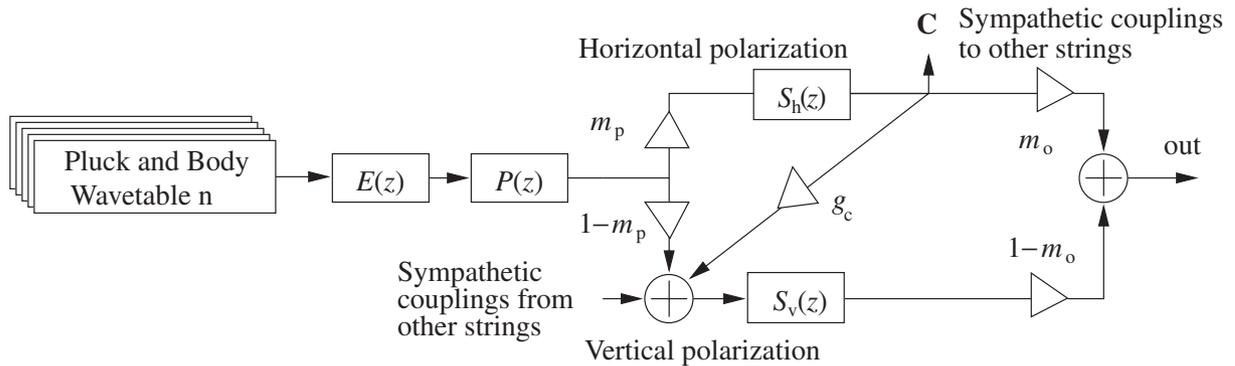


Fig. 2. A dual-polarization string model for the acoustic guitar including sympathetic couplings (Karjalainen et al., 1998). $S_h(z)$ is horizontal and $S_v(z)$ vertical submodel. C is matrix of sympathetic couplings, m_p and m_o control the coupling of the submodels, $P(z)$ is a plucking point filter, and $E(z)$ pluck tone control filter for wavetable excitation variation.

nance modes that amplify and color the instrument tone. It is shown in (Karjalainen & Smith, 1996) that a straightforward digital filter for high-quality simulation of the body transfer function is computationally expensive, for example an FIR filter of order 1000 or higher is needed. By structuring the body response into separate low-frequency modes and mid-to-high frequency reverberation, possibly using multirate techniques, the computational load can be reduced (Välämäki & Tolonen, 1998).

In the commuted synthesis model the string response and body response transfer functions are swapped so that the original cascade *excitation* \rightarrow *string* \rightarrow *body* is commuted to *excitation* \rightarrow *body* \rightarrow *string* and finally consolidated to *excitation & body* \rightarrow *string*, where *excitation & body* is the consolidated excitation that can be implemented as a wavetable. Using wavetables for combined excitations makes the synthesis extremely efficient. However, with rapidly increasing processor power the filter implementations of the body, allowing better parametric control of excitation and body functions, will become more attractive.

2.1 A full-scale LTI guitar model

A full-scale synthesis model needs more than the simplified model of Figure 1b can provide. Figure 2 depicts a block diagram of a synthesis model for the acoustic guitar which includes the most important LTI features of the instrument (Karjalainen et al., 1998).

As the first extension, a guitar string vibrates in two polarizations, the horizontal one in the body-top plain, and the vertical one perpendicular to it. Both modes may typically have different attenuation rates and even slightly different frequencies of harmonics. This difference, when the polarization components are combined, leads to beating (amplitude variations) or dual-slope attenuation, which is important for the naturalness of sound. To implement this, the synthesis model includes two single-polarization models in parallel. The second important addition is the couplings between the strings, in order to implement sympathetic vibrations, i.e., the

vibration of a string may excite another string with common partial frequencies. The gain coefficients m_p , m_o , and g_c (part of sympathetic coupling matrix), combined with all other string parameters, allow for powerful variation of the sound features of the synthesis model.

In the model of Figure 2 the excitation of the string model is carried out using a set of consolidated wavetables. Different plucking styles are covered with different wavetables while minor differences are made with a pluck filter $E(z)$ which can shape the excitation sharper or smoother. Another detail is the plucking point filter $P(z)$ that models the comb filtering spectral effects that are due to the position of pluck excitation.

A full-scale acoustic guitar needs six strings of the type shown in Figure 2. With fast DSP processors as well as modern PCs and workstations the model can easily run in real time – even several guitars may be simulated on a fast machine, including the computation of parametric controls.

3 Model control protocols

When playing a real physical instrument, the sound varies in a rich way that should be simulated with proper sequences of a synthesis model's control parameters. Since the design of control and synthesis strategies are closely interrelated, some questions upon the control protocols are discussed here briefly.

There are many possibilities to define a set of control parameters and controlling them to approximate a desired sound result. One possibility is to use as *physical parameters* as possible, such as string length and tension. In true physical modeling this is the goal but for sound synthesis it may not be practical. Another possibility is to directly use the *DSP model parameters*, such as filter coefficients. This is also inconvenient when synthesizing from music-related concepts. The third possibility is to search for *logical parameters* from a player's or a composer's point of view. In practical sound synthesis the third approach, using of musically

Fig. 3. Musical excerpt with ENP expressions.

and instrumentwise logical parameters is the best motivated alternative.

3.1 Control protocols for the guitar model

The control strategies for acoustic guitar synthesis were studied first in (Jánosy et al., 1994). By proper mappings from sequencer data or directly from MIDI data from a keyboard or another controller it was possible to produce convincing examples of guitar music. The general problem of instrument-specific control languages/protocols remained. We have studied recently the problem more systematically to find a well balanced parameter set and control protocol, including also playing a guitar model from standard music notation (Laurson et al., 1999; Hiipakka, 1999).

3.1.1 Expressive Notation Package (ENP)

The use of notation in expressive control is motivated by the lack of adequate real-time controllers, familiarity with common music notation, and precision of control. The use of common music notation requires no special technical training which in turn makes it possible to use professional players to test and verify various physical models in a deeper way than before. While common music notation provides a good starting point in expressive control it must be augmented in several ways in order to gain satisfactory results. A well known problem is for instance the fine tuning of timing during performance. Thus the user should have efficient and precise tools that allow to modify the basic rhythmical information provided by the input score. Besides standard instrument specific expressions (pizzicato, staccato, left-hand slurs) the input score must also include non-standard expressions. Non-standard expressions are usually not shown in ordinary notation but it is anyway assumed that a good player adds them according to her/his taste and the character of the piece.

We have developed a new notation package (Expressive Notation Package, ENP) to control the model-based synthesis engine. ENP is a PatchWork (PW, (Laurson & Duthen, 1989, Laurson, 1996)) user library and it is written in Lisp and CLOS (Common Lisp Object System). ENPO's object-oriented approach makes the system extendible and open.

ENP resembles commercial notation packages since it requires no textual input. Besides a full set of standard notation facilities, ENP has user-definable extensions that allow efficient description of interpretation.

Expressions can be applied to a single note (such as string number, pluck position, vibrato, or dynamics) or to a group of notes (left-hand slurs, finger-pedals). Groups can overlap and they may contain other objects, such as break-point functions. The latter case is called a group-BPF. Macro expressions generate additional note events (tremolo, trills, portamento, rasgueado). Figure 3 gives an ENP example – measures 25–28 from “Madroños” for guitar by Federico Moreno Torroba – which includes both standard instrumental expressions and nonstandard ones.

ENP allows fine-tuning of timing with the help of graphical tempo functions. In order to assure synchronization of polyphonic scores, all tempo functions are merged and translated internally into a global time-map (Jaffe, 1985). In addition to conventional accelerandi and ritardandi, the user can apply special rubato effects (“give and take”) to a group.

Besides tempo functions, ENP supports user definable performance rules which allow to modify score information. Performance rules are used to calculate timing information, dynamics and other synthesis parameters. The calculation of the control information for the model-based synthesis engine is executed in two main steps. In the first one, the note information provided by the input score is modified by the tempo functions and ENP performance rules. In the second step, all notes of the input score are scheduled. While the scheduler is running, each note sends a special method to its instrument which in turn starts other scheduled methods which typically produce the final control data. These methods are responsible for creating discrete control data (such as excitation information) or continuous data (gain of the loop filter, filter coefficients, or other low-level data). If a note contains a macro expression (trill, portamento, rasgueado) then new note instances are created on the fly by instrument specific methods and each new note is inserted in the scheduler queue.

3.1.2 Low-level control

A systematic control parameter set for the acoustic guitar model is quite close to the DSP parameters used in Figures

1 and 2. The main difference is the control of pitch that is coded to loop delay to yield the desired pitch. Pitch information can also be given by fret position that is natural for the guitar (ref. tablature). The loop filter parameters control the loop gain and lowpass frequency, effecting the frequency dependent attenuation rate of string vibration. These controls are desirable to map nonlinearly so that a logical number range, such as 0.0 . . . 1.0 or -1.0 . . . 1.0 can be applied. This principle is used with several other parameters. For details of the parameter set, see (Hiipakka, 1999).

The control protocol is the next question to be solved. Inside a closed system it is not a problem but when transmitting data between different systems and software it must be somehow standardized. Bytecode protocols such as MIDI are compact and efficient but not flexible and self-explaining. Textual protocols may be more open and readable and more easily extensible if well designed. The overhead of parsing text to machine instructions adds overhead but with modern fast processors this is not a problem unless transmitting over a limited bandwidth channel.

An interesting textual protocol was developed by Hiipakka (Hiipakka, 1999) where the syntax is of form

```
address/operation[?x1[?x2]]
```

Here `address` is a form compatible to network addresses in the Internet. It may consist of the network part, an orchestra, a section of the orchestra, an instrument, and a string. Thus an orchestra of instruments may be distributed in the network. The `operation` activates a function of the string or instrument, including (optionally) parameters. In the simplest form plucking a guitar string may be called as

```
guitar1/string1/pluck 0.5
```

This assumes that all necessary parameters except pluck gain (velocity), including pitch, have already been set. Value 0.5 in the above form is pluck gain (corresponds to velocity in MIDI).

Another interesting principle is to apply Extensible Markup Language (XML) for describing a control event structure. XML is expected to become an important portable format for many kinds of documents. In XML the information is expressed in tagged structures of elements and attributes, such as

```
<EVENT TIME="2.53">
  <INSTR>guitar1</INSTR>
  <STR>string1</STR>
  <WTABLE>normal1</WTABLE>
  <FRET>12</FRET>
  <PLUCK>0.5</PLUCK>
</EVENT>
```

An XML parser reads the textual document structure, and from the parsed structure a synthesis application can synthesize it. A well designed XML protocol is relatively easily readable also by humans.

Another direction is taken in streaming audio applications based on the MPEG-4 bytecode control of virtual sound systems (Vercoe et al., 1998). This is developed to support good real-time behavior in systems where data transmission capacity may be limited.

4 Types of non-LTI behavior

Nonlinear and time-variant physical systems appear in a multitude of forms that are difficult to classify in a few categories. If an LTI system f is defined as to conform to the requirements of superposition and linear scaling

$$f[ax_1(t) + bx_2(t)] = af[x_1(t)] + bf[x_2(t)]$$

for input signals $\{x_1(t), x_2(t)\}$ and being shift-invariant so that properties don't change in time, then all other systems are considered as non-LTI. Since probably no physical system is absolutely LTI, minor deviations from strict LTI property are linearized if possible. This has been the practice also in model-based sound synthesis in order to develop first-order approximations for physical systems.

The conceptually simplest forms of non-LTI behavior are memoryless nonlinearities. They can be approximated by mathematical functions, lookup tables with optional interpolation, or by techniques such as feedforward neural nets. Memoryless nonlinearities are found useful in the approximation of reed behavior in wind instruments (Cook, 1992; Välimäki et al., 1992). For example air jet oscillation in the flute can be approximated with a sigmoidal type of function $y = \tanh(x)$.

A general type of nonlinearity with memory can be considered as a nonlinear filter. Among techniques to approximate an arbitrary nonlinear filtering are neural net based filters, i.e., neural nets including delay elements (Luo & Unbehauen, 1997), and Volterra filters (Schetzen, 1980). Distributed effects of nonlinearity have been studied, e.g., in (Tassart et al., 1997, Msallam et al., 1997) and (Välimäki et al., 1999b; Tolonen et al., 1999). Pointwise distributed amplitude limiting nonlinearities are discussed in (Rank & Kubin, 1997). In this article we are interested in a special type of passive nonlinear physical behavior that is discussed in more detail below.

From a DSP implementation point of view nonlinearities are almost always problematic due to some of the following reasons:

- *Computational complexity.* All nonlinear functions – except square-law and point discontinuities such as absolute value and half-wave rectification – tend to be much slower to compute than linear operations, unless special hardware is used. A big difference is for example between linear digital waveguides and such ones where nonlinearity is distributed. In the former case distributed linear effects can be consolidated into a few points, but nonlinear operations cannot in general be commuted and consolidated. This may mean orders of magnitude difference in computational cost.

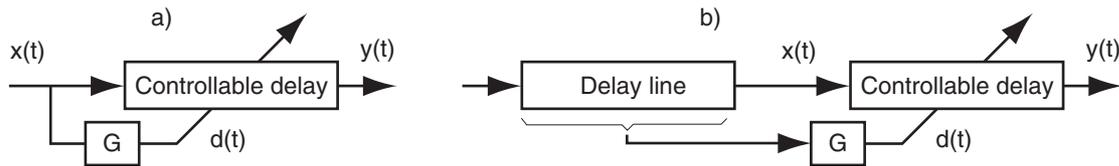


Fig. 4. Controllable delays as approximations for passive nonlinearity: (a) Input signal is used through transform G to control delay and (b) delay control signal comes from another part of the system, in this example it is computed from a distributed time-varying property of a string delay line.

- *Aliasing.* Signals in DSP are considered to be bandlimited according to the sampling theorem. Most nonlinear operations can create new frequency components, i.e., harmonics or intermodulation products that have frequency above the Nyquist frequency. This means that aliasing of these components happens, mirroring them back to the audio band with undesirable effects that may easily appear audible.
- *Other problems.* In some cases nonlinearities may imply delay-free loops (Borin et al., 1997) that have traditionally been understood as non-computable. There are ways to work around this but the efficiency problem may be encountered again. An inherent problem with nonlinearities is almost always how to estimate and approximate them for complex physical phenomena. This will not be discussed in this article.

4.1 Passive nonlinearities

A very common type of nonlinearity in musical instruments is such one which redistributes energy between resonance modes, but itself is passive, i.e., no energy is created or dissipated by the nonlinearity itself. This phenomenon, in relation with physical modeling of musical instruments, was introduced in (Van Duyne et al., 1994; Pierce & Van Duyne, 1997) and since then similar ideas have been developed further for example in (Välimäki et al., 1998; Välimäki et al., 1999b; Tolonen et al., 1999; Tolonen et al., 2000).

The fact that makes passive nonlinearities necessary and critical is that such phenomena typically appear inside feedback loops or matrices, whereby stability of the overall system is crucial. Typical memoryless nonlinearities are automatically out of question since their energy transfer function does not meet the requirement. For example Volterra filters are based on power series expansions where the output tends to grow rapidly when the input level gets large enough.

A systematic theory for signal processing implementations of passive nonlinearities does not exist yet. What is probably needed is first to study it from the energy behavior point of view and formulate it using continuous-time signal processing theory. Then this should be interpreted for band limited DSP techniques, also paying attention to the fact that the nonlinear aliasing mentioned above can make the discrete-time implementation non-passive although the continuous-time implementation were perfectly passive.

So far the best idea from the energy passivity point of view is to use a digital filter with allpass characteristics where the filter parameters are changed according to input or other control signal at the time moment when the energy of the filter is zero. Such first-order allpass filter solution was proposed in (Van Duyne et al., 1994). The problem that makes this deviate from an ideal one is that the filter coefficient cannot in a discrete-time implementation be changed exactly at zero value of the state variable. Thus it yields an approximation only.

If we relax somewhat more the requirement of absolute passivity, the idea of (Van Duyne et al., 1994) may be generalized further. In Figure 4 controllable delays are shown where the delay parameter is controlled by (a) input signal or (b) some other signal from the system. In the continuous-time case the output $y(t)$ for input $x(t)$ and delay control $d(t)$ is

$$y(t) = x[t - d(t)]$$

Note that for sinusoidal input and control signals this can be interpreted as phase modulation or frequency modulation so that this element has a resemblance to traditional FM synthesis.

The delay modulation element of Figure 4a can be approximated by an allpass filter or FIR-type fractional delay filter, such as a Lagrange interpolator (Laakso et al., 1996). Allpass filters have the advantage of a flat magnitude response but with parameter changes they create transients that take time to decay due to the recursive structure. There are techniques to eliminate transients (Välimäki & Laakso, 1998) although this makes the realizations more complex. FIR approximations, including Lagrange type fractional delays, do not generate transients although a change in filter parameter may of course result in a discontinuity of the output signal. Due to the transient-free behavior we have used Lagrange interpolators in the nonlinear modeling examples described below.

5 Tension modulation nonlinearity

A general type of physical nonlinearity that can be found in most string instruments, at least when playing loud, is due to tension modulation in the vibrating string. Any deviation from equilibrium makes the string longer which increases the tension. A commonly found effect due to this is a higher pitch of a note after pluck which then approaches the nominal pitch

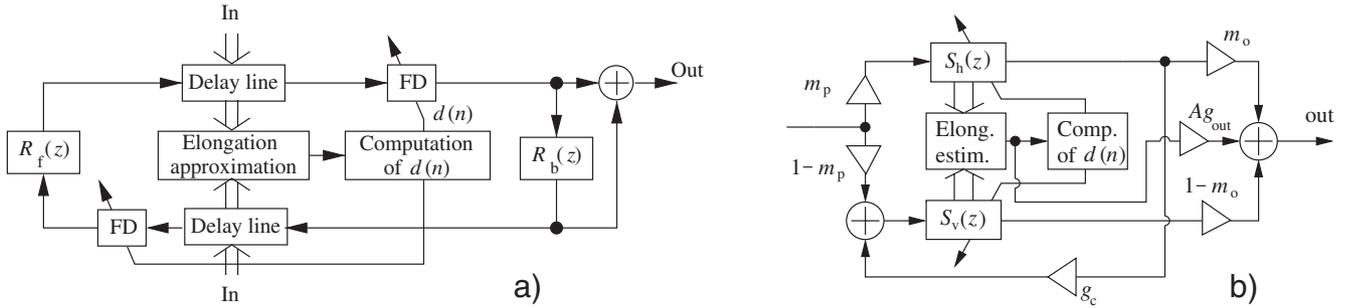


Fig. 5. Digital waveguide models for strings with tension modulation nonlinearity: (a) basic two-delay-loop model (Välämäki et al., 1999b; Tolonen et al., 2000) and (b) dual-polarization model and longitudinal tension coupling to output (kantele) (Välämäki et al., 1999a).

value. In some instruments, such as the Finnish kantele, the tension modulation may show very radical effects on the timbre of the instrument (Karjalainen et al., 1993a; Välämäki et al., 1999a) as discussed below.

Tension modulation depends essentially on the elongation of the string during vibration. Elongation may be expressed as the deviation from the nominal string length ℓ_{nom}

$$\ell_{\text{dev}} = \int_0^{\ell_{\text{nom}}} \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial x}\right)^2} dx - \ell_{\text{nom}}$$

where y and z are the two transversal polarization displacements of the string and x is the spatial coordinate along the string. Tension F_t along the string is linearly related to the elongation ℓ_{dev} as

$$F_t = F_{\text{nom}} + \frac{ES\ell_{\text{dev}}}{\ell_{\text{nom}}}$$

where F_{nom} is the nominal tension corresponding to the string at rest, E is Young's modulus, and S is the cross-sectional area of the string.

In a linear case the propagation speed of the transversal wave is $c_{\text{nom}} = \sqrt{F_{\text{nom}}/\rho_{\text{nom}}}$, where ρ_{nom} is the linear mass density along the string. When we assume that the longitudinal wave propagation speed, the linear mass density, and the tension are approximately spatially constant, we may write the propagation speed of the transversal wave as

$$c = \sqrt{\frac{F_t}{\rho}} = \sqrt{\left(\frac{\ell_{\text{nom}} + \ell_{\text{dev}}}{\rho_{\text{nom}} \ell_{\text{nom}}}\right) \left(F_{\text{nom}} + \frac{ES\ell_{\text{dev}}}{\ell_{\text{nom}}}\right)}$$

where ρ is linear mass density of the vibrating string given by $\rho = \rho_{\text{nom}} \ell_{\text{nom}} / (\ell_{\text{nom}} + \ell_{\text{dev}})$. The equation implies that c depends on the elongation ℓ_{dev} of the string. This in turn implies that the string vibration is not strictly speaking periodic. Thus, we use the term effective fundamental period to refer to a short-time average value of the period.

5.1 Digital waveguide modeling of tension modulation

The deviation of the delay-line length (in samples) can be approximated from the digital waveguide model as follows

$$\frac{L_{\text{dev}}(n)}{2} = \sum_{k=0}^{L_{\text{nom}}/2-1} \sqrt{1 + [s^+(n,k) + s^-(n,k)]^2} - \frac{\ell_{\text{nom}}}{2}$$

where $s^+(n,k)$ and $s^-(n,k)$ are the *slope* signals traveling in the delay lines, L_{nom} is the nominal delay-line length, and n and k are the discrete time and space indices, respectively. While this formula could be readily implemented, it can still be simplified. When we assume that $[s^+(n,k) + s^-(n,k)]^2 \ll 1$, we may develop a Taylor approximation of the square root function. When the second and higher-order terms are excluded, this yields

$$L_{\text{dev}}(n) = \sum_{k=0}^{L_{\text{nom}}/2-1} [s^+(n,k) + s^-(n,k)]^2$$

Now we see that the string tension modulation can be simulated by controlling a fractional delay filter with a *power-like* signal L_{dev} which is a sum of the pair-wise squared sums of the delay-line signals. For more details of the derivation, see (Tolonen et al., 2000). Figure 5a illustrates a digital waveguide string model with tension modulation according to the rules given above. The elongation approximation block computes the slope squared sum in order to control time-varying fractional delays. The model can be simplified to a single-delay-loop version (Välämäki et al., 1999b). In a fixed-length string the slope squared summation in space can be approximated by leaky integration in time at the end of the string. Another way to reduce computational complexity is sampling at sparse points for slope squared sum estimation.

6 Modeling of the kantele and the tanbur

The effects of tension modulation nonlinearities are audible in practically all string instruments when playing loud. In some specific instruments these phenomena are particularly prominent. We have studied two such cases in more detail: the Finnish *kantele* (Karjalainen et al., 1993a; Välämäki et al., 1999a) and the Turkish *tanbur* (Erkut et al., 1999).

The kantele is an old instrument with 5–40 fixed length strings and a sound box. Two special properties are found that shape the sound characteristics of the instrument: (a) one end of the string is terminated with a knot around a bar and (b)

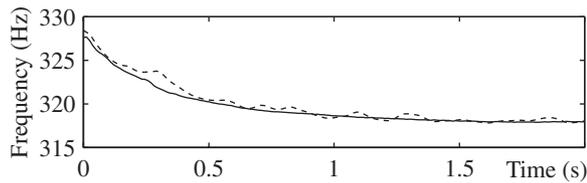


Fig. 6. Variation of fundamental frequency in a kantele tone as a result of fortissimo pluck (solid line) and in a synthetic tone (dashed line).

the other end is around a tuning peg directly without any supporting bridge. The first property means that the effective length of the string is different in vertical and horizontal polarizations, thus resulting in rich beating and “liveliness” of harmonics of the sound. The second property (no bridge) results in a strong coupling of the longitudinal tension modulation force through tuning peg to the sound box. Thus even harmonics are strong although in a fully linear model they should not exist when plucking a string in the middle. Also, energy transfer from a mode to another and between polarizations takes place. Figure 5b depicts a modification of Figure 5a where the dual-polarization behavior ($S_h(z)$ and $S_v(z)$, cf. Fig. 2) are taken into account as well as the tension modulation coupling to sound box through coefficient Ag_{out} (Tolonen et al., 1999; Välimäki et al., 1999a).

An audible effect, due to tension modulation, is the variation of fundamental frequency after the pluck event, especially in a fortissimo pluck. Figure 6 plots the measured f_0 of a real kantele signal as a function of time (solid curve) and the corresponding curve from a simulation model (dashed line). The dual-polarization model of Figure 5b can also simulate the beating of harmonics in an aurally adequate way.

The Turkish tanbur (Erkut et al., 1999) is another plucked string instrument of interesting nonlinearities. The tanbur is a long-neck representative of the oldest group of the lute instruments. It has a hemispherical body with a thin shell top plate and no sound hole, a violin-like bridge, and paired strings with unusual tuning scheme. Normally only a pair of strings is plucked and four other strings work as resonant strings.

As analyzed by Erkut (Erkut et al., 1999), the tanbur strings exhibit audible effects due to nonlinearities. The tension modulation phenomenon is found to introduce several percent of pitch drift when plucked hard and noticeable drift with moderate plucking. Nonlinear mode coupling in string vibration is shown by analyzing the amplitude envelopes of harmonic components. A good indication of nonlinear coupling is when a harmonic has initially very low level (depending on pluck position) and it starts to gradually acquire energy increasing its vibration level, until it decays along with other modes.

Similar nonlinear effects may be found by careful analysis in other string instruments as well. If they are audible, they should be included in high-quality model-based synthesis algorithms.

7 Summary

An overview of some recent developments in model-based sound synthesis techniques is presented in this article, primarily related to plucked string instrument synthesis. First an advanced linear model for the six-string acoustic guitar and its control have been discussed. Nonlinear string models, especially with passive nonlinearities, have been discussed with examples given for instruments where the tension modulation nonlinearity is inherently prominent.

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