Physical Modeling of Plucked String Instruments with Application to Real-Time Sound Synthesis*

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An efficient approach for real-time synthesis of plucked string instruments using physical modeling and DSP techniques is presented. Results of model-based resynthesis are illustrated to demonstrate that high-quality synthetic sounds of several string instruments can be generated using the proposed modeling principles. Real-time implementation using a signal processor is described, and several aspects of controlling physical models of plucked string instruments are studied.

0 INTRODUCTION

Physical modeling of musical instruments has become an increasingly active field of research in musical acoustics and computer music. The term physical modeling refers in this case to the mathematical or computational simulation of sound production mechanisms of musical instruments.

Traditionally musical sound synthesis, such as FM synthesis or waveshaping, has tried to achieve a desired waveform or spectrum [1], [2]. In sampling or wavetable synthesis, digital recordings of acoustic sounds are edited and processed for resynthesis. Although sampled tones may sound perfect, convincing synthesis of musical instruments has been difficult since one sample corresponds to a sound played by one instrument in a certain way. A small change in the playing technique would demand the synthetic sound to be resampled.

The approach taken in physical modeling can be divided into two classes:

1) Mathematical modeling of physical principles
2) Design of model-based sound synthesizers.

The former approach is typical for physicists who aim at understanding the acoustics of musical instruments. The latter approach—the one we have taken—is more practical, but a comprehensive understanding of the physics of the musical instruments to be synthesized is still needed.

Three different viewpoints are of paramount importance when designing a physically based synthesis technique. First, essential features of the physics of a musical instrument have to be studied carefully. Second, the properties of the human auditory system need to be considered because it is the ear that finally judges whether the synthetic sound is satisfactory or not. In practice a physical model can be constructed by simplifying the underlying physical principles. Here the knowledge of hearing can be of help: features that are not perceptually relevant need not be simulated precisely, which leads to simplification. The third point of view is that of a DSP engineer. The model should be computable in real time, preferably on a commercial (signal) processor. Thus the model should be easily and efficiently programmable.

In this paper we show how synthesis models for plucked string instruments, such as guitars, the banjo, the mandolin, and the kantele, can be constructed following these principles. In our approach the model for a single string is based on refinements of the Karplus–Strong (KS) algorithm [3], which is a simple computa-

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nitional technique for synthesizing plucked string sounds. As reported by Jaffe and Smith [4], this algorithm is a simplification of a physical model for a vibrating lossy string, which is based on the solution of the wave equation. Later, Smith generalized this approach to what is nowadays called digital waveguide modeling [5]–[7].

We have further improved these principles in our model, such as with a continuously variable digital delay line using Lagrange interpolation for adjusting the length of the string [8] and a simple yet precisely tuned low-pass filter—called a loop filter—to bring about the frequency-dependent damping of a string [9]. The model incorporates the influence of the body in an exceptionally efficient way [7], [9]–[11]. Additions to the basic string and body model are discussed in order to include beat effects inherent in string vibration and sympathetic couplings between strings [4], [11], [12].

After the model structure has been designed, an estimation method is needed for calibrating the values of the model parameters. A particular subproblem is the estimation of the coefficients of the loop filter. This problem was first addressed by Smith in the early 1980s [12], [13]. He studied several parametric methods for matching the loop filter and also proposed some nonparametric methods, such as the use of deconvolution. In [14] a time–frequency representation was applied to the analysis of musical sounds, and the loop filter was designed based on those data. In [15] the same approach was used in the analysis, but instead of using the KS model, a pair of poles was assigned to each sinusoidal component. The technique was applied to the synthesis of the piano, but was reported to have been tested for guitar tones as well.

We have obtained prototype frequency responses for the loop filters by means of short-time Fourier analysis and envelope tracking of the harmonics. A first-order all-pole filter is matched to the analysis data using a weighted least-squares design. Smith has independently developed a very similar analysis method [16]. Thereafter the input signal for the model can be extracted from a recorded string instrument sound using inverse filtering. This approach has also been proposed in [12], [14], [15]. The residual can be truncated or windowed, and the resulting short-duration sequence can be fed into the synthesis model to produce an approximation of the original signal. The parameters of the model can be modified, and yet a very natural sounding signal will result.

This paper presents an overview of our recent work on model-based sound synthesis of plucked string instruments. Our contribution has been to introduce high-quality real-time synthesis for the acoustic guitar [8], [11] and for several other instruments. The work includes refinements of modeling techniques, such as fractional delay filtering, body modeling, estimation of the model parameters, implementation techniques, and methods for controlling the physical models.

The organization of the paper is as follows. An overview of the plucked string instruments that are studied in this paper is given in Section 1. The principles of modeling string instruments are discussed in Section 2. Two basic models for a vibrating string and signal processing techniques such as Lagrange interpolation are briefly described. The basic models are extended by introducing, for example, a dual-polarization string model. The modeling of the body of a guitar is considered from three points of view—digital filter approximation, a principle based on commutativity of linear systems, and a hybrid where some resonances of the body are explicitly modeled and a processed input signal includes the rest of the body response. Section 3 concentrates on the analysis techniques that can be used for estimating the values of the model parameters. Examples of synthesis results are reported in Section 4. Real-time implementation techniques are considered in Section 5. In Section 6 control aspects of the physical models of plucked string instruments are discussed. Finally, conclusions are drawn and directions for future work are given in Section 7.

1 OVERVIEW OF PLUCKED STRING INSTRUMENTS

The history of guitarlike plucked string instruments spans to antiquity. Cultural and geographical differences have led to the evolution of a large family of string instruments over a long period of time. In this paper we have concentrated on the analysis and model-based sound synthesis of six example cases of the plucked string instrument family:

1) The modern classical nylon-string acoustic guitar
2) The flat-top steel-string acoustic guitar
3) The electric guitar
4) The banjo
5) The mandolin
6) The kantele

All these string instruments exhibit some special characteristics, which are to be taken into account when designing model-based sound synthesizers. The goal in this research was, however, not only to create sound synthesis methods, but also to use physical modeling as a tool in the research of different string instruments. A short overview of the string instruments featured in this paper is given in this section. A more detailed analysis of the acoustics of some of these instruments can be found in [17], for example.

1.1 Acoustic Guitar

Analysis and modeling of the classical acoustic guitar has been presented in previous papers [8], [9], [11], [18], but as an extension to guitar modeling we measured and modeled the behavior of the steel-string acoustic guitar. The main difference in the acoustics of these instruments is caused by two facts: 1) the material of the strings and 2) the plucking method. Steel-string guitars typically have crossed bracing in the soundboard due to the higher tension of the strings [18]. The use of a plectrum instead of the finger as the excitation method results in a stronger pluck and a brighter tone. These features can be taken into account in a physical model.
1.2 Electric Guitar

The main difference between acoustic and electric guitars is in the body of the guitar. The solid-body construction, as the one we measured (Fender Stratocaster), radiates very little sound from the body itself or the strings. Our model-based approach to the electric guitar does not cover the effects of magnetic pickups or nonlinearly behaving amplifiers. Instead we focused on the behavior of the string vibration in order to design a physical model for the plain electric guitar.

The three-dimensional plot in Fig. 1 exhibits the sound behavior of an electric guitar. The third string was plucked while other strings were damped. The neck pickup was used for the recording. It can be seen that due to the lack of body resonances the attack part is very well behaved. The decay of the harmonics is also smooth, that is, there is no nonlinear behavior. The signal was analyzed using the short-time Fourier transform (STFT) technique [19], which is discussed in Section 3.2.

1.3 Mandolin

The mandolin is a string instrument used widely in folk and bluegrass music in western countries. It features eight steel strings, which are tuned in four pairs. The sound of the mandolin is brighter and decays faster than that of the acoustic guitar. The pairs of equally tuned strings usually result in beating due to a minor difference in the tuning. The STFT-based spectral analysis of one string of the first string pair of the mandolin plucked with a plectrum near the bridge is discussed in Section 4.2 (see Fig. 27). The remaining strings, including the other string of the string pair, were carefully damped. The quite rapid decay of the harmonics as well as a linear behavior of the strings can be observed.

1.4 Banjo

The banjo differs from the classical acoustic guitar mainly in the construction of the instrument body. The five strings have been coupled via a metal bridge to a drumlike resonating plate. The time-varying spectrum of a banjo tone is depicted in Fig. 2. The figure shows the response after the first string was plucked with a finger pick at a distance of 140 mm from the bridge while other strings were damped. The resonances of the body can be observed in the first 200 ms of the tone. The impulse response of the soundboard has been found to be quite long. This must be taken into account in the excitation signal of the synthesis model (see Section 3.5). There is also two-stage decay behavior in the third harmonic, which may result from mode summation or coupling effects.

1.5 Kantele

The kantele is an ancient Finnish instrument that features 5 to 40 strings. Fig. 3 shows the five-string kantele that has been used in the analysis. There are two special characteristics in the kantele that are not found in other string instruments [20].

1) There are very strong beats in harmonic envelopes.
2) A prominent second harmonic exists due to longitudinal vibration effects.

Both effects were found to be due to the unusual way the strings have been terminated. The beats are generated when the horizontally and vertically polarized vibration components are superimposed since there is a clear difference in the effective string length in the two polarizations (see Section 2.6.2). This is due to the knot termination around the metal bar [Fig. 4(a)]. The strong second harmonic comes from the longitudinal tension variation that bends the tuning peg [Fig. 4(b)] and radiates from the soundboard proportionally to the square of the transversal string displacement.

Fig. 5 shows a three-dimensional STFT analysis result of a kantele tone. Note that the second harmonic is almost 10 dB louder than the first harmonic in the attack part, but it attenuates at a faster rate. Strong beating can be observed in higher harmonics.

1.6 Instrument Measurements

In order to find relevant analysis data for model-based synthesis we conducted accurate and thorough measurements of the chosen string instruments. Professional mu-
sicians and high-quality instruments were used. The ba-

sic measurements consisted of recording single notes
played on each string at several fret positions. Both fin-
ger picking and plectrum picking were used as the string
excitation method for instruments such as the electric
guitar and the steel-string guitar. Typical playing styles,
such as strumming, vibrato, and glissando, were also
recorded to obtain information for the control of the
models.

All measurements were carried out in a large anechoic
chamber using Brüel & Kjær microphones and pream-
plifiers. The measurement data were first stored on a
DAT recorder at 44.1-kHz sampling rate and 16-bit
quantization, giving a theoretical signal-to-noise ratio
of about 96 dB. In the second phase the data were trans-
ferred onto a hard disk using the QuickSig software
[21] designed at the Laboratory of Acoustics and Audio
Signal Processing of the Helsinki University of Tech-
nology, but only read and write pointers are updated. This may
reduce the computational load by several orders of mag-
itude as compared to a shift register, where the data
are actually moved.

2 GENERAL MODEL FOR A STRING

INSTRUMENT

A string instrument can be divided into functional
blocks, as illustrated in Fig. 6. The sound sources of the
instrument are the strings, which have been connected to
the body via the bridge and which also interact with
each other. The sound radiates mainly through the body.
The vibrating strings themselves act as dipole sources
and radiate sound inefficiently. First we concentrate on
the strings.

2.1 Basic Model for a Vibrating String

The general solution of the wave equation for a vibrat-
ing string is composed of two independent transversal
waves traveling in opposite directions (see, for example,
[17]). At the string terminations the waves reflect back
with inverted polarity and form standing waves. The
losses in the system damp the quasi-periodic vibration
of the string. The system is assumed to be linear, and
thus all losses and other nonlinearities may be
lumped to the termination and excitation or pickup
points. This is because we are only interested in the
output of the system and not in the excitation amplitudes at
arbitrary points inside the system. Thus we can combine
all the linear operations, namely, delays and filters, be-
tween the input and output points (see, for example,
[6]). The string itself can then be described as an ideal
lossless waveguide, which is a bidirectional discrete-
time delay line [22], [6]. This system may be modeled
using a pair of delay lines and a pair of reflection filters
$R_e(z)$ and $R_f(z)$, as depicted in Fig. 7. Each of these
filters includes the reflection function of the correspond-
ing termination of the string and the dissipative and
dispersive contribution due to the string material. A loss-
less delay line can be implemented computationally ef-
ciently as a circular buffer where the data do not move,
but only read and write pointers are updated. This may
reduce the computational load by several orders of mag-
nitude as compared to a shift register, where the data
are actually moved.

![Fig. 5. STFT analysis of kantele tone. (Fourth open string,
fundamental frequency 349 Hz.)](image)

![Fig. 3. Kantele, a traditional Finnish string instrument.](image)

![Fig. 4. String terminations of kantele. (a) Termination at metal bar, causing beat effect. (b) Termination at tuning peg, creating
lonitudinal nonlinearity.](image)
An efficient structure for a string model is obtained by commuting one of the delay lines with one reflection filter. Then the delay lines may be combined into a double-length line and the filters into a single one that can be expressed as

\[ H_i(z) = R_i(z)R_f(z) \]  

(1)

The transfer function \( H_i(z) \) is called the loop filter. This formulation is in practice equivalent to the model of Fig. 7 when a comb filter \( P(z) \) is cascaded with the string model to cause the filtering effect due to the plucking position [4]. The main difference is the delay between the excitation point and the output, but this is insignificant and can be compensated if necessary. This modified system, which we call a generalized KS model, is illustrated in Fig. 8. A more detailed derivation and analysis are given in [23].

The filter \( P(z) \) is called a plucking-point equalizer, and its transfer function is

\[ P(z) = 1 + z^{-M} R_f(z) \]  

(2)

where \( R_f(z) \) is the reflection filter of one end of the string model and \( M \) is the delay (in samples) between the excitation points in the upper and lower lines of the original waveguide model (see Fig. 7). In practice \( R_f(z) \) in Eq. (2) need not be modeled very accurately since its magnitude response is only slightly less than unity at all frequencies. In practice the most remarkable difference between direct and reflected excitation is the inverted phase, and thus the filter can be replaced by a constant multiplier \( r_f = -1 + \varepsilon \), where \( \varepsilon \) is a small nonnegative real number. The contribution of a pickup point in an electric guitar can be incorporated into the model using a similar comb filter [24].

The fact that the finger or plectrum exciting the string has a finite (nonzero) touching width has been ignored in this model, and it has been assumed that the excitation acts at a single point. Widening the excitation point to an interval adds more low-pass filtering to the excitation. We can bring about this effect using a low-order excitation filter \( E(z) \), which models the dependence of the spectral tilt on the dynamic level of the pluck. Jaffe and Smith [4] suggested that a one-pole digital filter be suitable for this purpose. Furthermore the complex interaction between the finger and the string has not been modeled. Instead, the excitation has been supposed to be an event that can be modeled using linear operations. In reality, the finger may grab the string for a short while, causing nonlinear or linear, but time-varying interaction.

### 2.2 Length of String

The effective delay length of the feedback loop in the string model determines the fundamental frequency of the output signal. The delay length (in samples) can be computed as

\[ L = \frac{f_s}{f_0} \]  

(3)

where \( f_s \) and \( f_0 \) are the sampling rate of the synthesis system and the desired fundamental frequency, respectively. In general \( L \) is a positive real number, and thus implementation of the delay line calls for the use of a fractional delay filter, which we denote by \( F(z) \). It is a phase equalizer that brings about a controllable fractional delay. A first-order all-pass filter [4] and a third-order maximally flat FIR filter based on Lagrange interpolation [8] have been suggested as alternatives for the implementation of the fractional-length delay line in a string model.

We used Lagrange interpolation for fractional delay approximation. The filter coefficients \( h(n) \) of the Lagrange interpolator are computed as (see, for example, [25])

\[ h(n) = \prod_{k=0}^{n} \frac{D - k}{n - k}, \quad n = 0, 1, 2, \ldots, N \]  

(4)

where \( N \) is the order of the FIR filter and \( D \) the desired delay. Lagrange interpolation is equivalent to the maximally flat approximation (at \( \omega = 0 \)) of band-limited interpolation. The approximation error (and its \( N \) derivatives) is zero at \( \omega = 0 \) and negative everywhere else (unless \( d = 0 \)), that is, the Lagrange interpolator is a low-pass filter. The computational efficiencies of \( N \)th-order maximally flat all-pass and FIR fractional delay filters are compared in [26]. All-pass fractional delay filters are recursive and thus are prone to transient effects when their delay is changed. In [27], [28] a new method was introduced to suppress the transients in time-varying

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Fig. 6. Block diagram of general model for plucked string instrument.

Fig. 7. Waveguide model for vibrating lossy string.

Fig. 8. Generalized Karplus–Strong model consisting of string model \( S(z) \) cascaded with plucking-point equalizer \( P(z) \).

\[ x(n) \rightarrow P(z) \rightarrow y(n) \]
To conclude our discussion on the frequency dependence of the string model, let us express \( L(\omega) \) by means of the three filters discussed,

\[
L(\omega) = L_1 + \tau_p(\omega) + \tau_f(\omega) + \tau_d(\omega)
\]

where \( L_1 \) is the integer number of unit delays in the delay line, and \( \tau_p(\omega) \), \( \tau_f(\omega) \), and \( \tau_d(\omega) \) are the phase delays of the loop filter \( H(z) \), the fractional delay filter \( F(z) \), and the dispersion filter \( D(z) \), respectively. Fig. 9 illustrates the implementation of the string model \( S(z) \).

2.3 Loop Filter

In the original KS model the loop filter \( H(z) \) is a two-tap averager which is easy to implement without multiplications [3]. This simple filter, however, cannot match the frequency-dependent damping of a physical string. We feel that an FIR filter is not suited to this purpose since its order should be rather high to match the desired characteristics. Instead we suggest the use of an IIR low-pass filter for simulating the damping characteristics of a physical string. The same decision was made in [12]. From [17], [33] it can be concluded that a second- or third-order all-pole filter could be suitable for a loop filter. It is important to use the simplest satisfying loop filter since one such filter is needed for the model of each string. Also, the loop filter is not a static filter, but its coefficients should be changed as a function of the string length and other playing parameters. This is easiest to achieve using a low-order IIR filter. We found that a reasonable approximation may be obtained using a first-order all-pole filter,

\[
H(z) = g \frac{1 + a_1}{1 + a_1 z^{-1}}
\]

where \( g \) is the gain of the filter at 0 Hz and \( a_1 \) is the filter coefficient that determines the cutoff frequency of the filter. For \( H(z) \) to be a stable low-pass transfer function, we require that \( -1 < a_1 < 0 \). The numerator \( 1 + a_1 \) scales the frequency response (divided by \( g \)) at 0 to unity, thus allowing control of the gain at \( \omega = 0 \) using the coefficient \( g \). We require that \( |g| \leq 1 \).

Fig. 10 shows the magnitude response and the group delay of the loop filter \( H(z) \) [Eq. (6)] with three different values of coefficient \( a_1 \). These values have been chosen so that they represent cases found in practice. In this example \( g = 0.99 \). (A change in \( g \) merely shifts the magnitude response curve vertically.) Note that the group delay of the loop filter is very small in all three cases.

Fig. 11 shows the impulse response of the string model \( S(z) \) with the three loop filters of Fig. 10. Here the length...
of the delay line is \( L = 19 \). It is seen that the impulse response decays more rapidly as the magnitude of \( a_i \) increases.

2.4 Modeling the Body

From the viewpoint of signal processing, the body of the acoustic guitar and the transfer function from the bridge to a specific direction may be considered as a high-order linear filter. A set of transfer functions would be needed to simulate the directivity pattern of a string instrument [34]. In most physical models only one transfer function has been included. This simplified approach simulates the radiation of the sound to one point in front of the sound hole of the string instrument. This kind of model will thus at best produce a sound reminiscent of a recording of a musical instrument in a relatively anechoic room. In Section 2.5 we discuss different methods to incorporate directivity characteristics into physical models.

The transfer function from the bridge to the listener can be measured approximately by exciting the bridge with an impulse hammer and registering the radiated sound. Figs. 12 and 13 show the impulse response and the magnitude spectrum of the body of a classical acoustic guitar, respectively. The impulse response can be seen to be long enough so that the temporal envelope can be assumed to be perceptually important. The situation is comparable to the perception of reverberation in a small room. This implies that a digital filter approximation should not be designed exclusively based on magnitude-response criteria. The spectral envelope seen in Fig. 13 is relatively flat, but there are a large number of resonances starting from the lowest resonance around 100 Hz.

We have studied several digital filter approximations for modeling the body of the acoustic guitar [18]. An FIR filter model of the body response must be at least 50 to 100 ms long (more than 1000 taps) to yield good synthetic sound. Linear prediction (LPC) analysis suggests an all-pole filter model of order 500 or more. Both of these are computationally too expensive for real-time implementation using a single DSP processor. We also designed reduced-order IIR filters that approximate the frequency resolution of the human auditory system, but even these did not reduce the computational load enough without audible degradation of the sound quality. A more detailed discussion of the body modeling problem is given elsewhere [23], [35].

To overcome the inherently heavy computational load of the filter-based body models a novel method was invented [7], [9]–[11]. Let us consider the string instrument model of Fig. 14(a). The output signal \( y(n) \) of this system can be expressed as

\[
y(n) = e(n) * h(n) * b(n)
\]

where the asterisk denotes the operation of linear discrete convolution, \( e(n) \) is the excitation signal [that is, the impulse response of the excitation filter \( E(z) \)], and \( h(n) \) and \( b(n) \) are the impulse responses of the generalized KS string model \( H(z) = P(z)S(z) \) and the body, respectively. Fundamental properties of linear operators, such as associativity and commutativity, are valid for discrete convolution, and thus Eq. (7) can be rewritten as

\[
y(n) = x(n) * h(n)
\]

where we have defined

\[
x(n) = e(n) * b(n)
\]

Thus it is possible to swap the transfer functions of the generalized KS model and the body, and, in addition,
row-band linear-phase FIR filters, are magnitude spectra for second-order IIR filters at azi-
values, and design a second-order all-pole filter to represent Fig. 15 depicts the modeling of direction-dependent radiation of acoustic guitar. We came to the conclusion that even first- or second-order directivity filters give useful re-
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Fig. 15 depicts the modeling of direction-dependent radiation of acoustic guitar (in the horizontal plane) relative to the main-axis radiation. Shown in the figure are magnitude spectra for second-order IIR filters at azi-
muth angles of 90, 135, and 180°. The reference magnitude spectrum at 0° is assumed to be flat. The low-
pass characteristic is noticeably increased as the relative angle is greater. The measurement was carried out by exciting the bridge of the instrument using an impulse hammer and registering the reference response at 0° and the related response in various directions. The measured reference and the directional response were fitted sepa-
ately with first- or second-order AR models. A simple division of the models was performed to obtain the pole—zero directivity filter.

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It is clear that computational modeling of the detailed directivity patterns is beyond the capacity of real-time DSP sound synthesis. It is therefore important to find simplified models that are efficient from the signal pro-
cessing point of view and as good as possible from the perceptual point of view. In earlier studies we have con-
sidered three strategies [34], [37]:
1) Directional filtering
2) Set of elementary sources
3) Direction-dependent excitation.

Directional characteristics must be taken into account when model-based sound synthesis methods are used for sound generation in room acoustics simulation and other virtual reality applications [34], [36].

Plucked string instruments exhibit complex sound radiation patterns due to various reasons. The resonant-mode frequencies of the instrument body account for most of the sound radiation (see, for example, [17]). In string instruments different mode frequencies of the body have their own patterns, such as monopoles, di-
poles, or quadrupoles, and their combinations. The sound radiated from the vibrating strings, however, is weak and can be neglected in the simulation.

Another noticeable factor in the modeling of directivity is masking caused by (and reflection from) the player of the instrument. Masking plays an important role in virtual environments where the listener and sound sources are freely moving in a space.

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where the coefficient values are $b_0 = 0.2041$, $b_1 = -0.1431$, and $a_1 = -0.4389$.

When zooming to the details of the lowest resonance modes we notice, as described in [17], that the individual modes behave differently. To model such details, a relatively high-order directional filter is needed. It is important to notice that due to the critical-band frequency resolution, an auditory smoothing may be applied to the responses before directional filter estimation. This helps to reduce the order of the filter.

The use of elementary sources is based on the idea that the radiation pattern of an instrument is approximated by a small number of monopoles or dipoles. In general the method is computationally expensive if a large number of paths to the receiver is needed since each new elementary source adds a new set of path filters.

In the case of commutative excitation [such as the plucked string instrument model shown in Fig. 14(b)] the directivity filtering may be included in the composite excitation in a way similar to the inclusion of the (early) room response [7]. The problem with this method is that the same number of string model instances are to be run in parallel as there are directions to be included. This limits the number of simulated directions, for example, to the six main directions of the Cartesian coordinate system. Even then this approach is computationally very inefficient.

The foregoing considerations as well as our experiments have shown that the directional filtering technique is normally the most practical one. A first- or second-order filter approximation is often a satisfactory solution in a real-time implementation.

2.6 Extensions of the String Model

The string model can be extended in several ways to more carefully simulate a vibrating physical string. In the following we present three examples. For a more detailed discussion, see [23].

2.6.1 Sympathetic Vibrations

Sympathetic vibration is a phenomenon where some modes of a string are excited by the vibration of other strings primarily due to couplings through the bridge. It can be simulated by feeding a small fraction of the output of each string model to the other strings [4], [11]. This simplified technique does not take into account the frequency dependence of sympathetic couplings, but it still adds realism to the synthetic tones. Since there is feedback via all strings, the values of the coupling coefficients must be carefully adjusted so that they are small enough not to make the system unstable. Another and theoretically more correct way to incorporate the sympathetic vibrations is to represent the bridge by a separate filter that is common to all strings in a waveguide model of an instrument [7]. This approach adds, however, computational complexity to the overall model.

2.6.2 Dual-Polarization String Model

Physical strings vibrate in both the vertical and the horizontal directions. If the effective length of the string is not the same in the two polarization planes, the mixing of the two subsignals of slightly different frequencies creates beats in the sound. This has been found to be an important feature in the sound quality of the kantele [20], where the explanation for this effect is the peculiar way in which the strings have been terminated. In other string instruments, however, the effective length in the two directions can differ due to changes in the driving-point impedance of the bridge. This can be incorporated in the model by using two basic string models with different L for each string [11]. Fig. 17 illustrates a complete model, including dual-polarization string, wavetable excitation, and sympathetic couplings.

In the case of the mandolin, beating is caused by the fact that there are four pairs of equally tuned strings. This can be modeled using two separate string models for each pair.

2.6.3 Nonlinearities

In strings the polarization of vibration may change continuously in a complex manner since there are weak nonlinearities transferring energy between modes and
polarizations. In the kantele, bending the tuning peg at
one end of the strings has been shown to induce nonlinearity due to longitudinal forces [20]. This effect has
been simulated successfully in the following way. The
delay line signal is squared, then filtered using a leaky
integrator, and the result is added to the output of the
string model. This procedure boosts the even harmonics
of the signal.

In [38] a passive nonlinear filter was proposed for
producing effects similar to mode coupling in string
instruments. A first-order digital all-pass filter is attached
to the end of the delay line of the string model. The
coefficient of the all-pass filter depends on the sign of the
delay line signal. Another alternative is to use a
variable delay based on a Lagrange interpolator FIR
filter. This technique was found to be suitable for pro-
ducing synthetic signals that exhibit time-varying behavior
in their decay parts.

3 CALIBRATION OF MODEL PARAMETERS

The synthesis system includes three parts that com-
pletely determine the character of the synthetic sound—
the string model, the plucking-point equalizer, and the
input sequence. This implies that to calibrate the model
to some particular instrument it is needed to estimate
the values for the length of the delay line \( L \), the coef-
ficients of the loop filter \( H(z) \), the delay \( M \) and the par-
parameter \( r_\ell \) of the plucking-point equalizer, and the input
signal \( x(n) \). In this section the parameter estimation pro-
cedures that will extract these values are described.

3.1 Estimation of the Delay Line Length \( L \)

The delay length \( L \) (in samples) determines the funda-
mental frequency \( f_0 \) of the synthetic signal according to
Eq. (3) so that

\[
f_0 = \frac{f_s}{L}
\]

where \( f_s \) is the sample rate. For pitch detection we have
used a well-known method based on the autocorrelation
function. The short-term or windowed autocorrelation
function is defined as [39]

\[
\phi_k(m) = \frac{1}{N} \sum_{n=0}^{N-1} [y(n + k)w(n)y(n + k + m)w(n + m)], \quad 0 \leq m \leq M
\]

where \( y(n) \) is the signal to be analyzed and \( w(n) \) is a
window function (such as the Hamming window). An
estimate for the pitch is obtained by searching for the
maximum of \( \phi_k(m) \) for each \( k \).

Typical pitch contours of a guitar and a kantele tone
are shown in Fig. 18. The contours have been smoothed
using a three-point running median filter [40]. In both
cases the pitch decreases with time and approaches a
constant value after about 0.5 s. In the kantele the de-
crease of the fundamental frequency is quite substantial.
The problem now is to determine the best estimate \( f_0 \)
for the fundamental frequency in a perceptual sense. In
practice a good solution is to use the average pitch value
after some 500 ms as the nominal value, since it is
important to have a reliable pitch estimate toward the
end of the note. This improves the quality of synthesis-
tized tones.

Once the estimate \( \hat{f}_0 \) for the fundamental frequency
has been chosen, the effective length \( L \) of the delay line
can be computed as

\[
L = \frac{f_s}{\hat{f}_0} 
\]

Fig. 17. Overview of plucked string synthesis model, including dual-polarization strings and sympathetic coupling between
strings.

Fig. 18. Fluctuation of fundamental frequency. (a) Steel-string guitar tone. (b) Kantele tone.
When the loop filter has also been designed, we can subtract its phase delay from \( L \) and define the nominal fractional delay \( L_F \) as

\[
L_F = L - \tau_H(\omega_0) - \text{floor}[L - \tau_H(\omega_0)]
\]

(14)

where \( \omega_0 = 2\pi f_0 \). The delay \( L_F \) is used as the desired delay in the design of the fractional delay filter \( F(z) \). In practice the phase delay of this filter can be expressed as

\[
\tau_F(\omega) = M_F + L_F,
\]

where positive integer \( M_F \) is the integral part of the phase delay that depends on the order of the fractional delay filter and \( L_F \) is the fractional part.

### 3.2 Measuring the Frequency-Dependent Damping

Next we discuss how to measure damping of a string as a function of frequency. As discussed in Section 2.1, all losses, including the reflection at the two ends of the string, are incorporated in the loop filter \( H(z) \). In order to design a loop filter we need to estimate the damping factors at the harmonic frequencies of the sound signal. This is achieved using the short-time Fourier transform (STFT) [19] and tracking the amplitude of each harmonic.

The STFT of \( y(n) \) is a sequence of discrete Fourier transforms (DFT) defined as

\[
Y_m(k) = \sum_{n=0}^{N-1} w(n)y(n + mh)e^{-j\omega_n}, \quad m = 0, 1, 2, \ldots
\]

(15)

with

\[
\omega_k = \frac{2\pi k}{N}, \quad k = 0, 1, 2, \ldots, N - 1
\]

(16)

where \( N \) is the length of the DFT, \( w(n) \) is a window function, and \( H \) is the hop size or time advance (in samples) per frame. In practice we compute each DFT using the FFT algorithm. To obtain a suitable compromise between time and frequency resolution, we use a window length of four times the period length of the signal. The overlap of the windows is 50%, implying that \( H \) is 0.5 times the window length. We apply excessive zero padding by filling the signal buffer with zeros to reach \( N = 4096 \) in order to increase the resolution in the frequency domain. The spectral peaks corresponding to harmonics can be found from the magnitude spectrum by first finding the nearest local minimum at both sides of an assumed maximum. The largest magnitude between these two local minima is assumed to be the harmonic peak. The estimates for the frequency and magnitude of the peak are fine-tuned by applying parabolic interpolation around the local maximum and solving for the value and location of the maximum of this interpolating function (see [41] or [42] for details). The number of harmonics to be detected is typically \( N_h < 20 \) (for the acoustic guitar). The STFT analysis is applied to the portion of the signal that starts before the attack and ends after some 0.5–1 s after the attack.

Spectral peak detection results in a sequence of magnitude and frequency pairs for each harmonic. The sequence of magnitude values for one harmonic is called the envelope curve of that harmonic. A straight line is matched to each envelope curve on a decibel scale, since ideally the magnitude of every harmonic should decay exponentially, that is, linearly on a logarithmic scale. Measurements show that this idealized case is rarely met in practice, and many different kinds of deviations are common. It is possible to decrease the error in fit by starting it at the maximum of the envelope curve [43] and by terminating it before the data get mixed with the noise floor, such as after the decay of about 40 dB. As a result, a collection of slopes \( \beta_k \) for \( k = 1, 2, \ldots, N_h \) is obtained (see Fig. 19). The corresponding loop gain of the string model at the harmonic frequencies is computed as

\[
G_k = 10^{\beta_k/20}, \quad k = 1, 2, \ldots, N_h
\]

(17)

where \( \beta_k \) are the slopes of the envelopes and \( H \) is the hop size. The sequence \( G_k \) determines the prototype magnitude response of the loop filter \( H_k(\omega) \) at the harmonic frequencies \( \omega_k, k = 1, 2, \ldots, N_h \), as illustrated in Fig. 20.

The desired phase response for \( H_k(\omega) \) can be determined based on the frequency estimates of the harmonics. We do not, however, try to match the phase response of the filter. There are two principal reasons for this. First we believe that it is much more important to match the time constants of the partials of the synthetic sound than the frequencies of the partials, since rather small deviations from harmonic frequencies occur in the case of plucked string instruments. In case we wanted to resynthesize strongly inharmonic tones, the phase response of the loop filter should be considered. The second reason is that as we want to use a low-order loop filter, the complex approximation of the frequency response would not be very successful. Thus we restrict ourselves to magnitude approximation only in the loop filter design.

![Fig. 19. Straight-line fits (dotted lines) of envelope curves (solid lines) of six lowest harmonics of electric guitar tone.](image-url)
3.3 Loop Filter Design

It is known that our hearing is sensitive to the change of decay rate of a sinusoid, and in practice we measure the time constant of some 20 lowest harmonics with the intent of matching the frequency response of the loop filter \( H(z) \) to those data. Since we use a first-order loop filter, it is clear that there are more restrictions than unknowns in this problem, and only an approximate solution is possible.

In principle the loop filter should be designed based on an auditory criterion, but since that would be complicated, we decided to use weighted least-squares design. The error function to be minimized is

\[
E = \sum_{k=0}^{N_h-1} W(G_k) |H(\omega_k)| - G_k]^2
\]

where \( N_h \) is the number of frequency points where the loop gain \( G_k \) is approximated, \( \omega_k = k\omega_0 \) are the central frequencies of the \( N_h \) lowest harmonics, and \( W(G_k) \) is a nonnegative error weighting function. It is reasonable to choose a weighting function \( W(G_k) \) that gives a larger weight to the errors in the time constants of the slowly decaying harmonics since our hearing tends to focus on their decay. A candidate for such a weighting function is

\[
W(G_k) = \frac{1}{1 - G_k}
\]

We require that \( 0 < G_k < 1 \) for all \( k \), which is a physically reasonable assumption since the system to be modeled is passive and stable.

Let us denote the numerator of Eq. (6) by

\[
A = g(1 + a_i)
\]

and \( \hat{H}(\omega) \) with the numerator removed by

\[
\hat{H}(\omega) = \frac{H(\omega)}{A}
\]

Then Eq. (18) can be rewritten as

\[
E = \sum_{k=0}^{N_h-1} W(G_k) [\frac{A\hat{H}(\omega_k)}{G_k} - G_k]^2.
\]

The gain \( g \) of the loop filter at low frequencies can be chosen based on the loop gain values of the lowest harmonics. In many cases it is good enough to set \( g = G_0 \), whereas sometimes the average of two or three lowest loop gain values gives a better result.

The value for the coefficient \( a_i \) that minimizes \( E \) can be found by differentiating Eq. (22) with respect to \( a_i \). This yields

\[
\frac{\partial E}{\partial a_i} = 2A_0 \sum_{k=0}^{N_h-1} W(G_k) \frac{\partial [\frac{A\hat{H}(\omega_k)}{G_k} - G_k]}{\partial a_i}.
\]

By substituting Eqs. (6) and (19), we can write

\[
\frac{\partial E}{\partial a_i} = 2A_0 \sum_{k=0}^{N_h-1} W(G_k) \frac{\partial [\frac{A\hat{H}(\omega_k)}{G_k} - G_k]}{\partial a_i}.
\]

The aim is now to find the zero of this function. In practice we find a near-optimal solution in the following way. The value of the derivative is evaluated, and depending on the sign of the result, \( a_i \) is changed by a small increment, the derivative is evaluated again, and so on. After the derivative has reached a very small value, the iteration is terminated and the final value for \( a_i \) is used in the synthesis model. We have verified the convergence of this design procedure in practice by analyzing signals generated using the synthesis model.

The loop filter that was designed based on the analysis data yields the same filter parameters — within numerical accuracy — as were used in the synthesis. Also the match with natural tones has been found to be satisfactory in many cases. Fig. 20 illustrates the loop filter design for a typical kantele tone.

3.4 On Estimating the Plucking Point

It is well understood that when a string is put to vibration by plucking it, the sound signal will lack those harmonics that have a node at the plucking point (see, for example, [17]). However, in general the string is not plucked exactly at the node of any of the lowest harmonics, and since the amplitude of the higher harmonics is considerably small anyway, it is not possible to accurately detect the plucking point by simply searching for the lacking harmonics in the magnitude spectrum. An-
other practical problem may occur because of nonlinear behavior of the string. Namely, the amplitude of vibration of a weak harmonic can gain energy from other modes so that its amplitude begins to rise, reaching a maximum about 100 ms after the attack, and then begins to decay [44]. This can often be seen in the analysis of harmonic envelopes of the guitar. For these reasons we believe that a more comprehensive understanding of the effect of the plucking point can be achieved by studying the time-domain behavior of the string in terms of the short-time autocorrelation function. A frequency-domain technique for estimating the plucking point has been reported by Bradley et al. [45].

Estimation of the plucking position is an inherently difficult problem since a recorded tone can include contributions of several delays of approximately the same magnitude, such as early reflections from objects near the player, the floor, the ceiling, or a wall. To minimize the effect of these additional factors, we suggest performing the analysis of tones recorded in an anechoic chamber.

It is, however, not absolutely mandatory to estimate and model the effect of the plucking position. Its contribution can be left in the excitation signal obtained using inverse filtering, which is described in the following section.

### 3.5 Inverse Filtering

The input signal \( x(n) \) can be estimated using inverse filtering, that is, filtering the signal \( y(n) \)—which is now assumed to be the output of the model—with the inverted transfer function \( S^{-1}(z) \) of the string model. The transfer function of the string model shown in Fig. 9 can be expressed as

\[
S(z) = \frac{1}{1 - z^{-\Delta}} F(z) H(z). \tag{25}
\]

If \( S(z) \) had zeros outside the unit circle, the inverse filter \( S^{-1}(z) \) would be unstable. In that case, inverse filtering would not yield an acceptable result. Fortunately this is not a problem in practice, as can be seen by substituting Eq. (6) into Eq. (25). Inverting this equation yields

\[
S^{-1}(z) = \frac{1 + a_1 z^{-1} - g(1 + a_1 z^{-1})^z^{-\Delta} F(z)}{1 + a_1 z^{-1}} \tag{26}
\]

This technique is simple to apply, but since the order of the loop filter is low, the harmonics are not canceled very accurately using the inverse filter \( S^{-1}(z) \). The resulting \( x(n) \) may suffer from high-frequency noise. The low-order loop filter was chosen primarily since it is then efficient to implement the synthesis model. However, inverse filtering is an off-line procedure, where accuracy is much more important than efficiency. We could thus design a higher order filter to be used in the inverse filtering (see for example, [12] or [15]).

Another deficiency of the inverse filtering technique is that it does not take into account the nonlinear effects of the physical strings. This can be accounted for by using a time-varying inverse filter, which cancels the impulse response of the string. This filter can be designed based on the STFT analysis discussed above.

However, satisfactory results are obtained when using carefully chosen tones that behave well, that is, there are no strong nonlinear effects nor beating present. Fig. 21 shows the spectrum of a mandolin tone and the magnitude response of the inverse filter. The magnitude spectrum of the residual (the result of inverse filtering) is illustrated in Fig. 22. Note that in this case the harmonics have been canceled quite accurately.

### 4 SYNTHESIS OF PLUCKED STRINGS

The residual signals resulting from the analysis and inverse filtering of an original string instrument tone can be directly applied as the excitation to the string model, as described in Sections 2 and 3. The synthesis procedure is carried out in the following way.

1) Obtain the residual signal by inverse filtering,

2) Window the first 100 ms of the residual signal using, for example, the right half of a Hamming window.
3) Use the truncated signal as the excitation to the string model.
4) Run the string model using the parameters derived from the analysis.

The original and the synthesized signals presented and discussed in this chapter are downloadable as sound files (AIFF, WAV, and MPEG-1 layer II formats) using the World Wide Web [46].

4.1 Guitars

From the analysis and synthesis results of the electric guitar shown in Figs. 23 and 24 it can be seen that the length of the residual signal used in resynthesis can be reduced to about 50 ms without any significant loss of information. This is due to the lack of body resonances. The resulting sound is quite similar to that of the basic KS model excited with an impulse. It is clear that in this case physical modeling principles should be extended to various magnetic pickup combinations and amplifiers.

The analysis and synthesis results of the steel-string acoustic guitar were similar to those of the nylon-string acoustic guitar. The use of the plectrum instead of the finger in the string excitation results in a different-sounding residual signal, but the string model can be the same for both instruments.

4.2 Mandolin

The analysis and resynthesis of the mandolin are illustrated in Figs. 25 and 26. We notice that windowing and truncation of the residual signal have only minor effects, shortening the decay of the lowest body resonances. The resynthesized tone in Fig. 26(b) has been calculated using the first 100 ms of the residual and a single-polarization string model. The original mandolin signal shows a slight beating effect after the transient part, whereas the resynthesized tone has an exponentially decaying behavior. It is, however, almost impossible to distinguish the synthetic tone from the original one by ear. Using a dual-polarization...
string model with a very small beating effect would result in an even more natural tone. Figs. 27 and 28 depict the temporal characteristics of the six lowest harmonics of the original and the synthetic mandolin tones, respectively. It can be clearly seen that the transient part is identical in both plots. The decay of the harmonics is very similar but not identical.

4.3 Kantele

The use of the dual-polarization string model is essential in the case of the kantele. As shown in Fig. 5, there is significant beating in the signal, which clearly affects the sound quality. Modeling this dual-polarization behavior is quite straightforward with two separate string models, as was discussed in Section 2.6.2. Figs. 29 and 30 depict the analysis and synthesis results for a kantele tone. A dual-polarization string model was used for the synthesis. The nonlinear characteristics of the kantele have been resynthesized according to the principles presented in Section 2.6.3.

4.4 Banjo

In the analysis of the banjo we experienced that most of the characteristics of a banjo tone are included in the attack part of the signal. The distinctive sound of the banjo is retained in the residual signal after inverse filtering. Figs. 31 and 32 depict the results of the analysis and the synthesis.

In this example some of the harmonics of the original signal decay much faster than the first harmonic. The synthesis model with a first-order loop filter is not able to catch this phenomenon. For this reason the decay rate of some harmonics is too slow in the resynthesized signal. This can be seen by comparing Figs. 31 and 32. Apart from this slight difference, the overall sound quality of the synthetic signal is excellent.

5 REAL-TIME IMPLEMENTATION ON A SIGNAL PROCESSOR

The real-time synthesis models were implemented using a Texas Instruments TMS320C30 floating-point signal processor. This processor is capable of executing a maximum of 30 million floating-point operations per

Fig. 27. STFT analysis of mandolin tone. (Gibson model A, first open single string, fundamental frequency 649 Hz.)

Fig. 28. STFT analysis of resynthesized mandolin tone.

Fig. 29. (a) Original kantele tone. (b) Residual signal after inverse filtering.

Fig. 30. (a) Truncated residual signal. (b) Resynthesized kantele tone.
second (30 MFLOPS, 15 MIPS). Our experience was, however, that this limit cannot be reached in practice, except in special cases when register, pipeline, and memory conflicts can be totally avoided. Our hand-optimized assembly code taking up about 110 instructions per output sample for a single string was able to run six strings in real time with a sampling rate of 22 kHz, including host communication and parameter calculation. However, this was found to be adequate for producing excellent plucked string sounds since there is not much energy in the spectrum of these signals above 10 kHz.

An overview of the software and hardware environment for the real-time implementation of physical models is given in Fig. 33. Programs written in the QuickC30 environment [47] can be run on different hardware platforms without modification. All hardware-dependent details are hidden from the application by the use of specialized macros and functions. So far the QuickC30 system has been implemented on the Macintosh and the PC platforms. We use commercially available DSP boards.

Our developments include using a multiprocessor environment for running several instruments in parallel or for a more detailed simulation of the instruments, including nonlinear effects. We have experimented with a system containing two TMS320C31 processors, which are less expensive and slightly reduced versions of the TMS320C30, and we have built a more advanced system using TMS320C40 floating-point processors, which support multiprocessing in hardware [36].

The implementation of plucked string synthesis on a signal processor follows the principles of Fig. 9. Each substring (single polarization) consists of a ring buffer delay line with third-order Lagrange interpolation (for fine-tuning) and a loop filter. Table look-up is used for computing the interpolator coefficients from pitch information. Also the loop filter parameters have been table coded. String excitations are read from wavetables that have been constructed using inverse filtering techniques.

Most control parameters are MIDI-like, that is, they cover a number range from 0 to 127. The computation of the model parameters, such as the delay and loop-filter parameters and excitation filtering, is done as much as possible by table look-up since computationally expensive divisions, logarithms, and exponents are otherwise needed. (For control parameters, see also Section 6.)

The updating of model parameters is executed every 1 ms, which has been found fast enough in most transients and transitions. The reduced parameter update rate is necessary to keep the computational cost low enough, such as in the case of guitar synthesis.

### 5.1 Multirate String Model

A string instrument tone, like most natural signals, is a low-pass signal whose spectrum varies through time so that high-frequency components are damped faster than low-frequency components. Thus it would be advantageous to design a multirate synthesis model where the input sequence is fed in at a high sampling rate, but where the delay line and the loop filter \( H(z) \) would run at a rate considerably lower. This idea has also been mentioned in Smith [48, p. 50]. As a result, the attack

![Fig. 31. (a) Original banjo tone. (b) Residual signal after inverse filtering.](image)

![Fig. 32. (a) Truncated residual signal. (b) Resynthesized banjo tone.](image)

![Fig. 33. Software and hardware environment for real-time implementation of physical models.](image)
part of the synthetic sound will still have energy at high frequencies, thus preventing the sound to be too low-pass filtered.

Fig. 34(a) illustrates the idea of the multirate string model. In the feedback loop the signal is first decimated by a factor $K$, which is a small integer number, say, 2 or 3. In decimation, the signal is low-pass filtered using a linear-phase FIR filter $H_K(z)$, and every $K$th sample is retained. The delay line length $L$ has to be divided by $K$ to keep the fundamental frequency constant. Also the loop filter $H_L(z)$ has to be redesigned. The output of the loop filter is upsampled by the factor $K$ and added to the input sequence.

A more efficient version of the multirate model is presented in Fig. 34(b). Now two versions of the excitation signal are needed—the original $x_p(n)$ (the output of the plucking-point equalizer) and a decimated $\hat{x}_p(n)$. In this model the excitation signal $x_p(n)$ is not processed at all since the feedback loop is computed separately. The output of the feedback loop, which runs at a lower sample rate, is upsampled using a linear-phase interpolating FIR filter $H_K(z)$. The Lagrange interpolator does not work well in this case since its magnitude response error is unacceptable at high frequencies. An all-pass fractional delay filter is better suited to this case.

6 CONTROL OF THE STRING INSTRUMENT MODELS

In this section we examine the necessary environment for using the models as musical instruments. Though the ideas described herein are not restricted to physical modeling, we believe that model-based synthesis is superior to conventional methods when it comes to control of the synthesis. Most model parameters have a direct physical meaning, and thus adjusting them changes the sound in a predictable way. One disadvantage of model-based methods is that each instrument family has to be modeled and controlled differently. In order to get a convincing sound, one has to study the performance techniques used with the given instrument, and, in a way, the performer has to be modeled as well.

6.1 Problems Related to String Instruments

Most plucked string instruments have several strings, the length of which can be fixed (such as the kantele or the harp) or variable (as in the lute family). There are several problems of sound synthesis control that appear only with certain types of instruments [49]. Here we consider two of them—note transitions and string allocation.

Note transitions appear when the next note has to be played on the same string that was used for the previous note. Transitions are perceptually very important. The lack of transitions makes the sound artificial in a melodic context, even if the individual notes sound realistic. Correct simulation of note transitions depends on two conditions—the ability to control the synthesizer in a proper way and the ability to recognize when transitions are needed. Physical modeling synthesis produces convincing note transitions without any special effort when the parameters of a model are adjusted for a new note. However, special care must be taken for eliminating extra noises caused by parameter updating.

The string allocation mechanism is responsible for finding the proper sound generation (that is, string model) for a new note. This is straightforward for instruments with fixed strings, since each pitch can be played only on a single string. If the number of generators is less than the number of available pitches on the real instrument (as with certain types of the kantele or, for example, in the case of the piano), a voice allocation mechanism is needed [49]. Otherwise each string can have a separate sound generator assigned to it. In this case the algorithm is straightforward: select the generator with the given (fixed) pitch. There are no note transitions in the sounds of these instruments.

A problem appears with variable-length strings. To allow polyphony, several strings are used on a single instrument. The pitch ranges of individual strings usually overlap. Thus the pitch does not uniquely determine the string. On the real instrument the performer has to select the appropriate string for a given note. Since individual strings typically have different physical properties (thickness, mass, and tension), choosing a different one will result in a slightly different timbre. When the computer has to perform a piece of music, a string allocation mechanism is needed to simulate this decision. This mechanism is entirely different from the voice allocation. It is needless to say that string allocation determines note transitions as well.

The simplest string allocation mechanism tries to as-

\[ x_p(n) \rightarrow H_K(z) \rightarrow H_L(z) \rightarrow y(n) \]

Fig. 34. (a) Multirate synthesis model where feedback loop is computed at lower sample rate $f_s/K$, with $K$ being the decimation factor. (b) Equivalent system, where original input signal $x_p(n)$ is fed directly to output and a decimated input signal $\hat{x}_p(n)$ is used in feedback loop.
sign the lowest string that can play the given pitch to
the note. A more elaborate method would be allocating
the strings so that the movement of the left hand would
be minimized. This can be accomplished easily if it is
possible to preprocess the score. However, this latter
method cannot be used with real-time control. As a com-
promise, nearly real-time control can be used [49].

6.2 MIDI and Intelligent Synthesis Control
MIDI has several caveats when used with complex
sound synthesis such as physical modeling. In principle,
one could use many different controller messages for
detailed control over the synthesis. This approach, how-
ever, has severe drawbacks. It is rather difficult to con-
trol many parameters manually and there are many pa-
terameter combinations that simply do not make a sound.
Another problem is that MIDI glues together the pitch
set and note triggering into a single “note on” message.
These events are clearly separated in time on the variable
source instruments. Therefore when such sound is syn-
thesized, these control events should be separated to
create natural note transitions.

In spite of these problems it would be convenient to
use existing MIDI equipment with new synthesizers.
We suggest a new approach for controlling complex
synthesis with MIDI: let the control interface—which
maps the performance events into control events—be
more intelligent. By intelligent we mean that it should
work reasonably, even if not all details are given explic-
tely. The missing information can be deducted from a
knowledge base created by studying the performance
techniques of the given instrument and analyzing the
impact of different playing techniques on the sound.
This approach has an additional advantage—it is much
easier to play an intelligent instrument than a complex
but dumb synthesizer.

6.3 Simplifying the Control Problem
The usual timbre space has three independent param-
eters—pitch, level, and timbre. These parameters are
too general for controlling specialized synthesizers such
as those based on physical models. However, during the
performance of a musical piece the timbre will probably
not exit a bounded subspace stretched by the possible
sounds of, for example, a given acoustic guitar within
a given performance style. Naturally there will be differ-
ent subspaces for different instruments and performance
styles, but the main point is that the performer can
choose the adequate subspace a priori. It may also be
necessary to allow shifting from one subspace into an-
other during the performance if the conditions change
substantially.

For moving around inside of one of these subspaces we
use fewer parameters—pitch and something we call
“expression.” Choosing pitch as the primary parameter is
motivated by our interest in traditional Western music,
which is entirely based on the twelve-tone scale and
where the most important property of a note is its pitch.
In our approach all internal parameter calculations are
based on the desired pitch.

While pitch is a well-defined property—in practice it
 corresponds roughly to the fundamental frequency of
a note—expression cannot be uniquely defined. Still,
everyone “feels” what it means. Although these feelings
are not necessarily the same, it can be agreed that a
low value of expression means something like colorless,
passing, or soft, whereas a high value can be interpreted
as emotional, bright, emphasized, or loud. Moreover the
interpretation of the expression seems to be closely
related to the performance style. The realization of this
led to the idea of combining the timbre and the mapping
of pitch and expression to the actual synthesis param-
eters into performance styles, like classical guitar, blues
guitar, or flamenco guitar.

Reducing the control parameters to only two drasti-
cally reduces the complexity of the control interface,
and it still allows human interaction. For more detailed
control, the attack part of the notes, which carries the
most important indication of the playing technique,
could be controlled separately. On the keyboard these
expression controls could be assigned to the aftertouch
and key velocity, respectively.

6.4 Overview of QuickMusic
QuickMusic is the musical sound synthesis control
package of the QuickSig system (see Fig. 33) and is
written entirely in Common Lisp/CLOS. It contains a
platform-independent sequencer, which uses a Lisp-syn-
tax musical notation and a real-time MIDI interface.
Originally the package had been developed under Macin-
tosh Common Lisp (MCL). Recently we have ported it
(together with other parts of the QuickSig environment)
to the PC platform under Allegro Common Lisp for
Windows.

6.5 The Lisp Sequencer
The Lisp sequencer can be used to quickly create test
note sequences or even demo pieces. For describing a
score a Lisp-like musical notation is used, allowing easy
parameterizing of the individual notes by keyword pa-
rameters. The sequencer is object oriented and thus eas-
ily expandable.

The score is processed in four phases—parsing, per-
forming, exploding, and playing. During the parse phase
a high-level object description of the note sequence is
generated. The perform phase was included to allow
automatic performance of the piece using performance
rules. This part has not been exploited so far. The ex-
plode phase generates (usually several) low-level control
events from the notes. Finally the built-in scheduler
plays the piece on the DSP hardware in real time.

6.6 MIDI Control Interface
The most important elements of QuickMusic are the
different control interfaces that map the input MIDI
events (such as key on/off, wheel movement) to the
actual synthesis parameters. They were added to allow
interactive performance and easy experimenting with rea-
time synthesis algorithms. As an additional advantage
of using MIDI, an external MIDI sequencer can be used
to record and replay performances.

A platform-independent MIDI message dispatch system has been built on top of the low-level interface. Message dispatching is accomplished automatically by the CLOS method dispatching mechanism, since the generic method process-midi-message is specialized on both the instrument class and the type of MIDI message. The MIDI control interfaces are implemented as different instrument classes.

6.7 Operating Modes

The interactivity of Lisp makes it easy to experiment with different control strategies. We have developed several different modes, partly to overcome the limitations posed by MIDI itself or to facilitate playing "guitarlike" on a normal keyboard. The keyboard, of course, by no means can replace a guitar controller (just as much as the synthesizer does not replace the acoustic guitar), but these special modes help getting the most out of the synthesizer in terms of fidelity and ease of playing.

6.7.1 Standard MIDI Modes

In order to be able to use existing MIDI files sequenced by others we have implemented the standard MIDI control modes as an option.

Mono Mode: The MIDI mono mode is often used with MIDI guitar controllers to send the performance data on separate MIDI channels for each string. This mode simplifies the MIDI implementation in the receiver. There is one sound generator per channel, and there is no need for voice allocation. Ambiguous pitches, which could be played on different strings, do not cause a problem. Pitch bend can be used individually on different strings without affecting other notes.

While the mono mode is perfectly suited for the MIDI guitar, it is difficult to use it with a keyboard. Although keyboard split could be used to send data on different channels, playing chords is quite difficult if not impossible this way.

Poly Mode: In the poly mode the data for all strings are received on a single MIDI channel, and the control interface has to assign the notes to available sound generators. There is a potential problem with instruments of the variable source type. Assigning a different generator to a new note will not result in a correct transition. String allocation is necessary for most string instruments.

6.7.2 Smart MIDI Modes

Smart MIDI modes are enhanced MIDI modes. While they accept MIDI events intended for normal use, they try to add more details to the sound, such as by splitting the "note on" to separate pitch set and note start events. Since setting the pitch happens before starting the note, this can only be achieved by delaying the start of a note by a small amount. Other enhancements, such as automatic fret noise generation, need even more delay. Allowing a relatively large (0.5-1 s) constant processing delay helps in both cases. When the synthesizer is driven by a sequence, the delay can be compensated by advancing the given track by the amount of delay.

Unfortunately this technique cannot be used in real-time performance.

6.7.3 Special Control Modes

We have experimented with advanced control modes for the guitar synthesizer. They help in creating a natural sounding guitar performance using a standard MIDI keyboard. Two basic styles have been investigated—solo playing and strumming.

Solo Guitar: Playing a guitar solo is slightly different from using the MIDI mono mode. On the guitar a solo is usually played on several strings. Therefore automatic string allocation is used. In the solo mode all special playing techniques that will be discussed later are available.

Rhythm Guitar: One difficulty of playing voiced guitar chords correctly on the keyboard is the relatively large internote separation of the chord members. To reduce this problem we have implemented an intelligent chord recognizer, allowing simplified chord fingering. The algorithm tries to fit different chords to the depressed keys and the most probable chord is selected (such as A, Am, A7). It is then revoiced for the guitar using look-up tables. When alternative voicings are possible (as in most cases), the position of the previously played chord can be used to select the chord in the closest position. The player can influence the selection of the chord position by using different octaves of the keyboard.

Each performance style can have its own set of chords, which is based on the most often used chords of the given style. Furthermore, styles can contain chords that use only the upper strings, leaving the lower strings free for playing bass notes, as in folk or Latin-American styles. In this case the bass notes can be played on another part of the keyboard using the keyboard split mode, or they can be played by using a special note-triggering mode. The notes are not triggered immediately by fingering a chord, but separate keys—one key per string—are used to pluck the strings afterward. This technique can be used for playing nice arpeggios as well.

With these techniques many common guitar styles can be played from the keyboard with little effort.

6.8 Mixing Control Modes

It is possible to switch between control modes during performance. This feature helps to play chordal solos interleaved with short runs. It is also possible to split the keyboard and define different control modes over the two ranges.

6.9 Special Techniques

In this section we discuss some special playing techniques not available on keyboard instruments. These techniques can be simulated with our intelligent control interfaces.

6.9.1 Hammer-On and Pull-Off

Hammer-on and pull-off are left-hand techniques that a guitar player sometimes uses to pluck a note. By striking a finger against a fret, one can set the string into
motion without plucking it with the right hand. This
technique can be used if the pitch of the hammered note
is higher than the previous one on the same string. It is
convenient for playing grace notes or fast passages. A
similar technique is fret tapping, used by many electric
guitarists, which allows playing very fast solos with both
hands on the fret board.

Pull-off is a complementary technique, which results
in a lower pitch. The left-hand finger releases the fret
used for the previous note, and at the same time it plucks
the string by pulling it a little sideways. Advanced play-
ers can play complete melodies without using their right
hand at all.

When hammer-on recognition is on, the control inter-
face assumes that the new “note on” that is played on
an already sounding string should be a hammer-on or a
pull-off, depending on the direction of the pitch change.
In the solo mode this can be accomplished by hitting
the next key before the previous one has been released
(similarly to the fingered portamento in the mono mode
on many synthesizers). In the poly mode the string allo-
cation algorithm suggests a hammer-on or pull-off when
it cannot allocate a new string with the given configura-
tion. In every case, releasing a key before the next one
is pressed ensures that the new note will be plucked.

6.9.2 Tremolo Picking

Tremolo picking is a fast up-down alternate stroke
plucking technique, typically used with the mandolin,
the bouzouki, or the balalaika. The string is not damped
manually between the individual strokes, but a new
pluck will automatically damp the previous note, creat-
ing a characteristic sound. This effect can be somehow
imitated on a normal keyboard by fast repetition of a
key. However, this way it is not possible to tell the
stroke direction, which is important in double-stringed
instruments. Moreover, the standard interpretation of
“note on” and “note off” would cut away the note in
between the repetitions, which sounds very unnatural.
Though the sustain pedal could be used to avoid this
artifact, as a result the new notes would be triggered
without damping the old ones, thus making a ringing
effect instead of the tremolo picking.

The use of the special string excitation keys (discussed
under Rhythm Guitar) automatically results in a correct
transition to the new note. Furthermore there is a sepa-
rate tremolo key which, when pressed, plucks the last
note again with a different pluck sequence, correspond-
ing to the opposite stroke direction.

6.9.3 Fingered Vibrato

Automatic vibrato sounds usually rather unnatural. It
can be replaced by mapping the keyboard aftertouch to
“pitch bend up” with a small pitch bend range. This way
expressive vibratos can be played easily by hand.

6.9.4 Muting the Strings

Unlike on the piano, on the guitar the strings are not
muted automatically. The guitar player can mute them
either by using his or her right hand or palm, or by
decreasing the pressure on the frets with the left hand
(a technique often used in country or beat strumming
styles). Both techniques result in a quick damping of
the string. Right-hand muting often causes an extra
sound when the palm hits the guitar body and the strings.
In contrast, left-hand muting usually does not introduce
extra noise, and it is often only partial, damping the
higher harmonics more than the fundamental.

In our system muting can be accomplished by stepping
on the sustain pedal. The same effect can be assigned
to a key. Partial muting—an often used effect with the
electric guitar—can be controlled with the expression
pedal.

6.9.5 Changing Pluck Position

Plucking a string close to the bridge causes a softer,
brighter, and sharper tone, while plucking toward the
neck makes a louder, mellower sound. Because of the
position of the right-hand fingers, the low strings are
usually plucked further away from the bridge than the
higher ones. Moreover, varying the pluck position has
an artistic effect by changing the overall timbre. On the
keyboard the position can optionally be mapped to the
key velocity (softer touch will move it closer to the
bridge) or to the modulation wheel.

6.10 Parameter Calculation

The calculation of the actual synthesis parameters—
delay-line length, loop filter coefficients, and fractional
delay filter coefficients—is based on two-dimensional
look-up tables indexed by the desired pitch and expres-
sion values. We are experimenting with an alternative
method that employs a neural network for implementing
the nonlinear mapping functions. Both methods use pa-
rameters derived from actual performances recorded pre-
viously and analyzed by a computer.

7 CONCLUSIONS AND FURTHER WORK

A physically based modeling technique for plucked
string instruments has been developed aiming at high-
quality real-time sound synthesis. The model for a vi-
birating string used in this method is based on a general-
ized KS model, which is constructed of digital delay
lines and linear filters. Many approaches to the modeling
of the body of string instruments have been studied, and
particularly successful results in terms of efficiency and
sound quality have been obtained by using an inverse
filtered input signal.

The parameter values of the synthesis model can be
calibrated based on the analysis of acoustic signals, and
thus the model can imitate the sound of a real instrument.
The analysis consists of pitch detection and short-time
Fourier analysis. Estimates for the length of the delay
line of the string model and the coefficients of the loop
filter can be obtained based on these analysis data.

The synthesis model accurately reproduces the attack
part (the first 100 ms or less) of the signal and approxi-
mates the average decay rate of the harmonics. Since
these two aspects are of great importance in the recogni-
tion of a musical instrument, the individual synthetic signals are often indistinguishable by ear from the original ones.

Principles presented in this paper can be applied to many plucked string instruments. We have investigated model-based synthesis of the acoustic guitar, the steel-string guitar, the electric guitar, the banjo, the mandolin, and the kantele. Nonlinear extensions which are required for imitating the beating of harmonics were also discussed. Real-time implementation of the synthesis model using a signal processor was studied, and a novel multirate implementation technique was developed. Several aspects of controlling the physical models of plucked string instruments were studied.

Future work in modeling plucked string instruments includes the design of a more sophisticated inverse filtering technique that would take into account the time-varying character of each harmonic. Furthermore a method to estimate the plucking point is needed. The plucking point of the string has to be determined to cancel its effect from the analyzed signal.

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9 REFERENCES


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