Fractional Delay Digital Filters

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***Abstract* — Fractional delay digital filters (FDDF) can be used for implementing discrete-time systems which include noninteger delays, i.e., delays that are not multiples of the sampling period. In this paper, we discuss the use of FIR-type FDDFs for implementing junctions of digital waveguides. As examples of useful applications, the fractional delay extension of the Kelly-Lochbaum tube model and a side branch placed at an arbitrary point of a digital waveguide are presented.**

I. INTRODUCTION

Fractional delay digital filters (FDDF) are digital systems containing delay elements that are not integer multiples of the sampling interval. Although any digital filter can be turned to an FDDF by replacing the unit delay elements by fractional delays, this principle is most naturally employed in digital time-domain models for physical systems, like in vocal tract models for speech synthesis [1] or waveguide models for musical instruments [2]. In these systems the finite sampling interval has proved to be a severe restriction.

In this paper, we discuss implementation techniques for *digital waveguides* [3] [4] which are digital systems simulating one-dimensional physical waveguides, e.g., electrical transmission lines, acoustical waveguides (narrow tubes or ideal strings), microwave transmission lines, or optical fibres. In digital modeling of these systems, the concept of fractional delay has turned out to be most useful. The resulting digital systems are called fractional delay waveguide filters (FDWF).

In the following discussion, the signals propagating in waveguides are assumed to be bandlimited and sampled according to the Nyquist theorem. Normally, the processing of this kind of signals can only take place at the sampling points. Our main goal is to show that by using FDDFs it is possible to get rid of time-domain limitations imposed by the finite sampling interval. In other words, we are aiming at achieving "virtually time-continuous" digital signal processing.

We have studied the implementation of FDDFs using FIR filters as interpolators. With this kind of interpolating filters, the value of a signal can be computed at any point within accuracy of certain approximation error. The transpose of the FIR interpolator can be used for adding a signal into a fractional point of another signal. This inverse operation of interpolation is called *deinterpolation*.

In principle, it is also possible to use digital allpass filters for implementing FDDFs [5] [6]. The use of nonrecursive FIR interpolators however offers some practical advantages over recursive filters.

After theoretical considerations we discuss practical methods for implementing arbitrary-length digital waveguides. With FDDF techniques, any number of digital waveguides can be connected to each other at an arbitrary point. As examples, we present fractional delay extension for the well-known Kelly-Lochbaum tube model [7] and a three-port junction where a side branch is connected to an arbitrary point of a digital waveguide.

II. IMPLEMENTATION OF DIGITAL WAVEGUIDES

Let us consider three basic elements of waveguide systems: 1) a single waveguide, 2) a waveguide junction (Kelly-Lochbaum scattering junction), and 3) a side branch. The elements are illustrated in their analog form in Fig. 1. The arrows illustrate signals propagating in two opposite directions in the waveguide. More complicated elements, like connections of four or more waveguides, can be defined similarly. These elements are needed when building a physical model for a real-world waveguide system.

The digital versions of the basic waveguide elements are shown in Fig. 2. Each waveguide is implemented as a bidirectional delay line. Here it is assumed that fractional delay elements are available so that the lengths of the delay lines or the points of connection, i.e., the placement of the

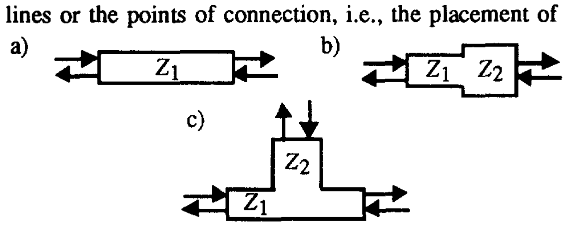

Fig. 1. a) A single waveguide, b) a waveguide junction, and c) a waveguide with a side branch.

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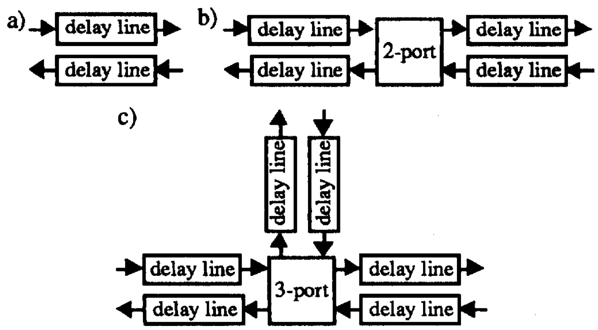

Fig. 2. Block diagrams for the basic digital waveguide elements: a) a digital waveguide, b) a connection of two digital waveguides via a two-port, and c) a connection of three digital waveguides via a three-port.

junctions between waveguides can be arbitrarily adjusted. The impedance of waveguides only affects the reflection coefficient of the scattering junction. The actual digital implementation of the waveguide junctions is discussed in more detail in Section IV.

III. Approximation and Implementation Techniques for FDDFs

A. *Ideal interpolation*

The ideal bandlimited interpolation between sampled signal values can be accomplished by using an ideal half-band lowpass filter. The frequency response of an ideal continuously adjustable delay element is

$$H(e^{j\omega}) = e^{-j\omega D} \tag{1}$$

where $\omega = 2\pi fT$ is the normalized angular frequency, $D \in R$ is the length of the delay as multiples of the unit delay, and T is the sampling interval. The magnitude response of the ideal delay element is identically one at all frequencies and its phase response is

$$\arg[H(e^{j\omega})] = -D\omega\,. \tag{2}$$

This corresponds to a linear phase filter. The phase delay – and also the group delay – of this delay element is thus equal to D, and the element passes all the frequency components of a signal with the same delay.

The inverse Fourier transform of the frequency response (1) is

$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{\sin[\pi(n-D)]}{\pi(n-D)} \tag{3}$$

where $-\infty \le n \le \infty$. The result (3) is a shifted and sampled version of the well-known sinc function ($\mathrm{sinc}(x) = \sin(\pi x)/\pi x$). If D is an integer, the impulse response of the delay element is zero at all sampling points except at $n = D$. When D is a fractional number, the impulse response has non-zero values at all index values n. Thus, the impulse response $h(n)$ of the delay element is a sampled version of the sinc function, which is infinitely long. Due to its infinite length, the impulse response corresponds to a noncausal filter, which cannot be made causal by finite shift in time. To produce realizable fractional delay elements, some finite-length approximation for the sinc function has to be used.

B. *Fractional Delay Approximation Using Lagrange Interpolation*

Several techniques have been introduced for digital filter approximation of a fractional delay [6]. *Lagrange interpolation* is a special case of a maximally flat approximation of the ideal frequency response (1). The Lagrange interpolator can be interpreted as an FIR filter, the coefficients of which can be computed from the formula [1] [5]

$$h_{D,N}(n) = \prod_{k=0,\,k\neq n}^{N} \frac{D-k}{n-k} \tag{4}$$

where N is the order of the FIR filter and D is to be chosen as

$$D = \frac{N-1}{2} + x \tag{5}$$

for minimum approximation error for even N, and x is the fractional part of the delay length.

Using (4), the interpolator can easily be designed, which makes the Lagrange interpolator the simplest FIR-type interpolator. Other FIR interpolators must in general be designed with more elaborate methods [6]. The Lagrange interpolator has best possible magnitude response at low frequencies, and the magnitude of its approximation error is largest at high frequencies. However, its magnitude response never exceeds unity, which is an important consideration when the overall system contains feedback. These advantages make Lagrange interpolation a good solution for applications where fractional delays are needed.

C. *FIR-Type Interpolation and Deinterpolation*

When an FIR filter is used for fractional delay approximation, the computation of input or output of the delay line at any noninteger point becomes natural. The interpolation out from a delay line is standard FIR filtering, i.e., a weighted sum of a finite number of adjacent samples. Like this it is possible – within limits of an approximation error – to compute the value of a sampled signal at any non-integer point D between samples. This is illustrated in Fig. 3a.

The input to a delay line can be implemented by the transpose of the FIR filter structure, where the signal value to be fed to a noninteger point of the delay line is weighted by each of the FIR filter coefficients and the products are added to adjacent values of the delay line. This is a practical way to

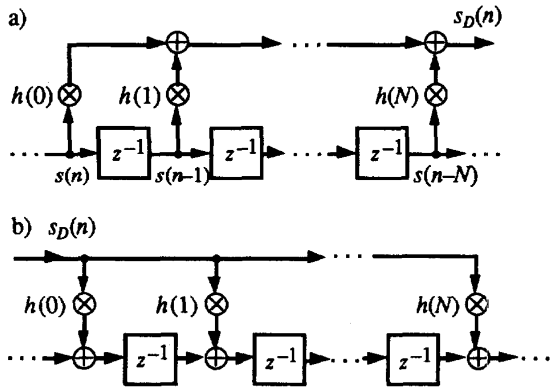

Fig. 3. FIR-type a) interpolation and b) deinterpolation.

insert an input to any point D of the delay line. The corresponding structure is shown in Fig. 3b. We call this operation *deinterpolation*.

Since deinterpolation can be realized by the transpose of the FIR interpolator, its transfer function is the same as that of the original FIR interpolator.

IV. Fractional Delay Waveguide Systems

In the following, we discuss fractional delay extensions of two waveguide elements, the two-port or Kelly-Lochbaum junction and the three-port junction.

A. The Fractional Delay Extension of the Kelly-Lochbaum Junction

The Kelly-Lochbaum (KL) tube model [7] is often used for simulating the human vocal tract or other one-dimensional waveguides. The area profile of the vocal tract is approximated by a chain of short uniform tubes. The KL model is a discrete-time implementation of the tube system. The computationally most expensive part of the tube model is the junction between tubes of different areas where scattering occurs. When tubes longer than one unit delay are used the number of junctions is decreased and the computational load is reduced. Using FDDFs it is possible to use tubes of arbitrary lengths.

In Fig. 4, a scattering junction of the traditional KL model for sound pressure waves is illustrated. In the fractional delay version of this model, the difference of the scattered signal in comparison with the unscattered signal (i.e., the propagating signal when the tube diameters are equal) is computed. The computation now consists of three parts: 1) interpolation of the signal out of the delay line, 2) computation of the scattering signal, and 3) deinterpolation of the scattered signal back to the delay line.

The scattering of sound pressure at a junction of two digital waveguides is described by a reflection coefficient r that is defined as

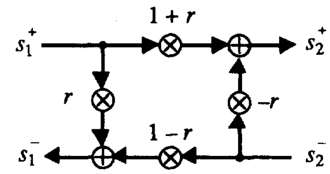

Fig. 4. A 2-port connecting two digital waveguides, like in Fig. 2b.

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{6}$$

where Z_1 and Z_2 are the impedances of the two adjacent waveguides. The impedances are assumed to be real. In the case of complex impedances, the reflection coefficient should be implemented as a digital filter instead of a real multiplier.

In Fig. 5, the computational structure for the fractional delay Kelly-Lochbaum junction is illustrated. First, the signal value $s^+(n-D)$ at an arbitrary point D in the delay line is estimated using an Nth-order FIR interpolation as

$$s_D^+(n) = \hat{s}^+(n-D) = \sum_{k=0}^{N} h(k)\, s^+(n-k) \tag{7}$$

where $h(n)$ is the impulse response of the FIR interpolator and $s^+(n)$ is the signal propagating in the upper delay line of Fig. 5. A similar equation can be written for the signal $s^-(n)$ propagating in the lower delay line. Equation (7) can also be written in vector form as an inner product

$$s_D^+(n) = \mathbf{h}^T \mathbf{s}_n^+ \tag{8}$$

where $\mathbf{h} = [h(0)\ h(1)\ h(2)\ ...\ h(N)]^T$ and $\mathbf{s}^+_n = [s^+(n)\ s^+(n-1)\ s^+(n-2)\ ...\ s^+(n-N)]^T$. The scattering signals $w^+(n)$ and $w^-(n)$ are, in the case of acoustic pressure waves, equal to each other and can be computed as

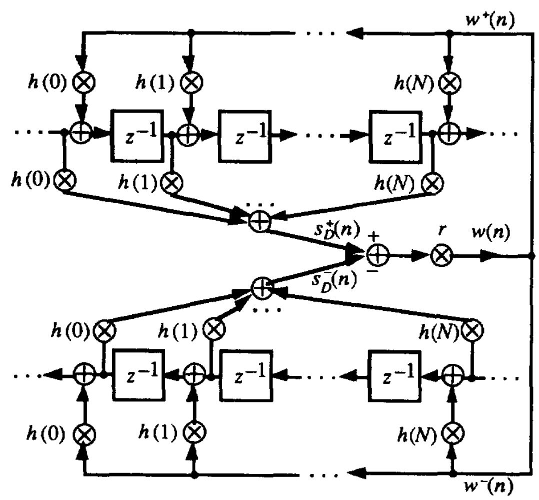

Fig. 5. The fractional delay Kelly-Lochbaum junction implemented using Nth-order FIR interpolation and deinterpolation.

$$w(n) = w^+(n) = w^-(n) = r \cdot (s_D^+ - s_D^-)$$
$$= r \cdot (\mathbf{h}^T\mathbf{s}_n^+ - \mathbf{h}^T\mathbf{s}_n^-) = r\,\mathbf{h}^T(\mathbf{s}_n^+ - \mathbf{s}_n^-). \qquad (9)$$

The signals after scattering are computed by deinterpolation, i.e, by adding the weighted copies of signals $w^+(n)$ and $w^-(n)$ to the original signals. This yields

$$s_2^+(n-k) = s_1^+(n-k) + h(k)\,w^+(n) \qquad (10)$$

$$s_1^-(n-k) = s_2^-(n-k) + h(k)\,w^-(n) \qquad (11)$$

for $k = 0, 1, 2, ..., N$. It is seen from (9) – (11) that the signal is in effect filtered twice with the transfer function of the FIR interpolator.

The fractional delay technique based on Lagrange interpolation yields good results in the low end of the frequency band, but in the upper end the delay is not as accurately approximated. The reason is that at high frequencies the phase delay of the Lagrange interpolator is zero or one. The fractional delay approximation is thus achieved only at lower frequencies. The approximation error can be interpreted so that the place of the junction depends on the frequency. In this case, the junction lies at a fractional point only at lower frequencies. At very high frequencies, the junction lies at the nearest integer point.

B. A Side Branch at an Arbitrary Point in a Digital Waveguide

Using the same principles as before, a fractional delay three-port junction, where a digital waveguide is connected to another as a side branch, can be constructed. The simplified data flow graph of the system is shown in Fig. 6. This is an approximate implementation of the ideal element shown in Figs. 1c and 2c.

The reflection coefficient r for the sound pressure signal is now computed as

$$r = -\frac{Z_1}{Z_1 + 2Z_2} \qquad (12)$$

where Z_1 is the (real) impedance of the digital waveguide and Z_2 is the impedance of the side branch (see Fig. 1c).

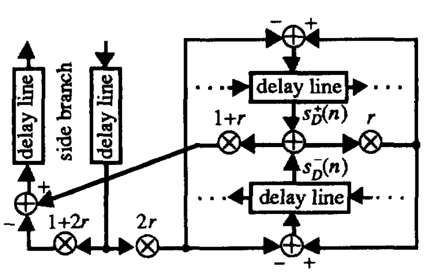

Fig. 6. A side branch connected to a fractional point of a digital waveguide. The input and output of the delay lines are computed using Nth-order FIR interpolation and deinterpolation.

C. Implementation issues for FDDFs

Fractional delay systems should not be implemented exactly as shown in Fig. 5. With this flow diagram we are only trying to give a clear view of the example system. Instead, e.g., the interpolation part of an FDDF should be implemented according to the last form of (9), because then only one FIR filtering per junction is needed.

When using FIR interpolation and deinterpolation, there must be a signal sample available in the delay for each filter coefficient. Consequently, placing a fractional delay junction near the end of a digital waveguide is difficult.

It is important to consider the causality of the FDDF system. The rule is that, when using several fractional delay junctions within one digital waveguide, the signal propagating in the delay lines must be read for each FDDF process before modifying the delay lines. After that all the interpolations, junction computations, and deinterpolations can be executed in parallel. Violation of this rule would cause causality problems in the overall system.

V. CONCLUSIONS

In this paper, we have shown how FDDF techniques can be employed for virtually time-continuous implementation of digital waveguide filters. By using appropriate interpolation methods, the constraints imposed by the finite sampling interval can effectively be eliminated.

These results are applicable to, e.g., articulatory speech synthesis and simulation of one-dimensional musical resonators, like strings or narrow tubes. Furthermore, FDDFs can be directly used with digital delay lines if the delay time needed is not an integer multiple of the sampling interval.

The proposed methods will be applied to real-time sound synthesis models for woodwind instruments. The fractional delay three-port junction is suitable for modeling the finger holes of these instruments. Another application is a nasal tract model in speech synthesis.

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