SIMULATION OF PLUCKED STRINGS EXHIBITING TENSION MODULATION DRIVING FORCE

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ABSTRACT

Recently, a nonlinear discrete-time model that simulates a vibrating string exhibiting tension modulation has been presented. This paper elaborates the previous model by taking into account two phenomena: 1) coupling of the tension modulation force to the instrument body and 2) feedback of the tension modulation force to the vibrating string. The two phenomena are investigated by analysis of plucked string instrument tones. Sound synthesis examples are available at www.acoustics.hut.fi/˜ttolonen/icmc99/ including tones by the proposed model, and by a linear model and a previous nonlinear model. The examples show that tension modulation coupling phenomena are important for the quality of synthesized tones and that the tones by the presented model are more natural than with the previous models.

1. INTRODUCTION

Plucked string instruments have been subject to extensive research in musical acoustics and physics-based sound synthesis. Although nonlinear behavior of vibrating strings is relatively well-known, the freely vibrating string has earlier been regarded as a linear system in modeling and sound synthesis [1, 2, 3]. Accordingly, linear discrete-time systems have been used in simulations. However, more elaborate models are required in order to include nonlinear phenomena which are in many cases critical for the naturalness and quality of the character of produced synthetic tones.

Recently, nonlinearities have been incorporated in string instrument modeling [4, 5], and a model has been proposed that includes tension modulation (TM) nonlinearity in the simulation of a vibrating string [6, 7, 8]. Tension modulation results from elongation of the string during its vibration; when the string is deviated from its equilibrium position, it is elonged and the tension along the string is increased. The TM nonlinearity is among the perceptually important nonlinear phenomena that is exhibited by every real string. It produces variation of the fundamental frequency and timbre changes caused by coupling of the harmonic modes.

The tension modulation also introduces a longitudinal force that acts on the bridge of the instrument. In some cases, this tension modulation driving force (TMDF) is strongly coupled to the body, and it thus has a significant effect on the quality of the tone. In addition, the TM force is coupled back to the vibrating string resulting in a mechanism for nonlinear coupling of the vibrations of harmonic modes.

In this contribution, we extend the previous TM string model to include a TMDF radiation mechanism. As a case study, we show how the TMDF is observed in analysis of the tanbur, a traditional Turkish instrument [9]. The tanbur features long and relatively loose strings coupled to a quasi-hemispheric body shell covered with a thin soundboard lacking a soundhole. Thus, the tension modulation is pronounced and the TMDF is both easily observable and perceptually important for the character of tanbur tones as shown by analysis of string and body vibration, and of recorded tones.

2. TENSION MODULATION IN PLUCKED STRINGS

Tension modulation depends essentially on the elongation of the string during vibration. Elongation may be expressed as the deviation from the nominal string length $\ell_{\rm nom}$ in meters [10]

$$\ell_{\rm dev} = \int_0^{\ell_{\rm nom}} \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2}\, dx - \ell_{\rm nom}, \qquad (1)$$

where y is the displacement of the string and x is the spatial coordinate along the string. Tension $F_{\rm t}$ along the string is linearly related to the elongation $\ell_{\rm dev}$ in meters and it can be expressed as [10]

$$F_{\rm t} = F_{\rm nom} + \frac{ES\ell_{\rm dev}}{\ell_{\rm nom}}, \qquad (2)$$

where $F_{\rm nom}$ is the nominal tension corresponding to the string at rest, E is Young's modulus, and S is the cross-sectional area of the string.

In the linear case, the propagation speed of the transversal wave is $c_{\rm nom} = \sqrt{F_{\rm nom}/\rho_{\rm nom}}$, where $\rho_{\rm nom}$ is the linear mass density along the string at rest. When

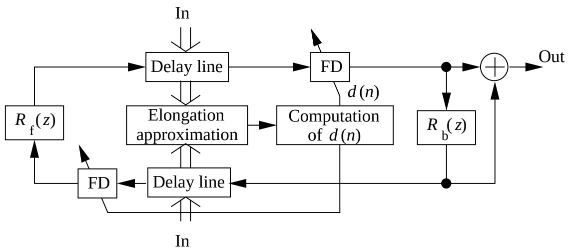

Figure 1: Dual-delay-line waveguide model of a string including tension modulation simulation [6, 7].

we assume that the longitudinal wave propagation speed is considerably larger than the transversal propagation speed, the linear mass density and the tension are approximately spatially constant and we may write the propagation speed of the transversal wave as

$$c = \sqrt{\frac{F_{\mathrm{t}}}{\rho}} = \sqrt{\left(\frac{\ell_{\mathrm{nom}} + \ell_{\mathrm{dev}}}{\rho_{\mathrm{nom}}\ell_{\mathrm{nom}}}\right)\left(F_{\mathrm{nom}} + \frac{ES\ell_{\mathrm{dev}}}{\ell_{\mathrm{nom}}}\right)} \tag{3}$$

where ρ is linear mass density of the vibrating string given by $\rho = \rho_{\mathrm{nom}}\ell_{\mathrm{nom}}/(\ell_{\mathrm{nom}} + \ell_{\mathrm{dev}})$. Equation 3 implies that c depends on the elongation ℓ_{dev} of the string. This in turn implies that the string vibration is not strictly speaking periodic. Thus, we use the term effective fundamental period to refer to a short-time average value of the period.

3. WAVEGUIDE MODELING OF TENSION MODULATION

The linear waveguide model of a plucked string [11] has been elaborated to include the nonlinear tension modulation simulation in previous studies [6, 7]. The implementation of tension modulation is based a *time-varying fractional-delay* structure [12]. Fig. 1 shows the dual-delay-line waveguide model with tension modulation. The delay-lines simulate the two traveling waves in the string. $R_{\mathrm{b}}(z)$ and $R_{\mathrm{f}}(z)$ are the reflection filters at the bridge and the fret, respectively. FDs are fractional delay filters [13] that are used for modulation of the delay line lengths. The input to the model is summed simultaneously to the all the elements in the two delay lines (the wide arrows in the figure).

In the elongation approximation block the elongation of the string is computed based on the slope variables that travel in the delay line. Using the elongation in samples, $L_{\mathrm{dev}}(n)$, the time-varying delay parameter is computed from the tension modulation as [7]

$$d(n) \approx -\frac{1}{2} \sum_{l=n-1-\hat{L}_{\mathrm{nom}}}^{n-1} \left(1 + A\right) \frac{L_{\mathrm{dev}}(l)}{L_{\mathrm{nom}}} \tag{4}$$

where L_{nom} is the nominal string length in samples, $\hat{L}_{\mathrm{nom}}$ is L_{nom} rounded to the nearest integer, $L_{\mathrm{dev}}(l)$ is the time-varying deviation, and $A = ES/F_{\mathrm{nom}}$. Methods for estimating the A parameter from recordings of plucked string tones are described in [7]. The time-varying part of the tension is given by $F_{dev}(n) = AL_{\mathrm{dev}}(n)$.

4. TENSION MODULATION FORCE COUPLING

In a previous study, an acoustical analysis of the tanbur, a Turkish long-necked lute, has been presented [9]. The TMDF coupling phenomena were analyzed by simultaneous investigation of the string vibration, acceleration of the soundboard near the bridge, and radiated sound when the string was plucked at its midpoint so that the initial excitation of even harmonics was small. The string vibration was detected using a magnetic pickup, and an accelerometer was attached to the soundboard beside to foot of the bridge. A high-quality condenser microphone was used to capture the radiated sound. The experiment was conducted in an anechoic chamber.

The TMDF coupling is most easily observed by analysis of the amplitude envelopes of the second harmonic of the three measured signals. In the linear case, plucking at the midpoint results in the lack of the second harmonic in the string vibration and thus also in the accelerometer signal and the radiated sound. When TMDF phenomena are presented, however, the second harmonic is strong in both the soundboard vibration and the radiated sound. This is expected since tension modulation is a second-order nonlinearity that creates a component at double the frequency of the first mode. Furthermore, since the string terminations are not completely rigid, TMDF is coupled back to the string so that the second mode begins to vibrate with an increasing amplitude.

Fig. 2 shows the amplitude envelopes of the second mode of the string vibration (solid line), the acceleration of the soundboard (dashed line), and the radiated sound (dash-dot line). The curves have been obtained using a short-time Fourier analysis. Note that the curves do not reflect the relative amplitude levels of the signals but they have been placed in the plot so that inspection of the curveforms is easy. The second mode of the string vibration exhibits a slowly rising amplitude envelope, as expected. Both the accelerometer signal and the radiated sound have second modes with sharp attacks that start to decay immediately after the attack. These results suggest that both the TMDF coupling to the body and to the vibrating string take place in the tanbur.

Fig. 3 shows in two steps how the tension modulation coupling can be included in plucked-string simulation. In the top block diagram, the TMDF coupling to the body via the bridge is taken into account by introducing a connection from the tension modulation to the string termination at the bridge (cf. Fig. 1). Note that since we have a force output at the bridge corresponding to the tanbur or the acoustic guitar, it is convenient to sum the TMDF signal with the force output

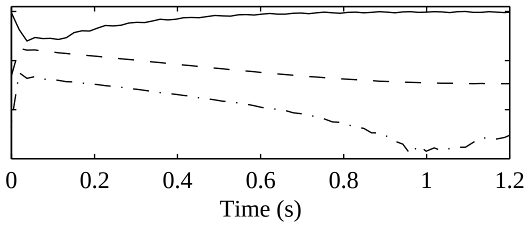

Figure 2: The amplitude envelope of the second harmonic of mid-plucked tanbur tone as detected in the string vibration (solid line), in acceleration of the soundboard (dashed line), and in the radiated sound (dash-dot line).

at the bridge. With this construction, the TMDF signal produces the sharp attack of the second harmonic if the virtual string is plucked at its midpoint. However, it does not affect the vibration of the string, i.e., the amplitude of the second harmonic in the string vibration remains small. The output coupling signal is given by $g_{\text{out}}F_{\text{dev}}(n) = Ag_{\text{out}}L_{\text{dev}}(n)$ parameter. Note that in the instrument the coupling is not so simple. The tension modulation may act in longitudinal, vertical and horizontal directions, depending on the bridge construction. If we wished to model this accurately, we would need to have separate transfer functions that model how the tension modulation components are radiated from the body. However, for practical sound synthesis purposes, the proposed model is sufficient to reproduce the effects caused by the coupling phenomena.

The block diagram on the bottom of Fig. 3 illustrates how the TMDF signal can be coupled back to the string. In this case, the TMDF signal is added into the delay lines at the string termination. The parameter g_{str} controls the coupling. Notice that while the model in top block diagram is guaranteed to be stable, the bottom block is potentially unstable due to the feedback path. While a strict stability analysis of the model including the tension modulation nonlinearity is a tedious task, the g_{str} parameter can in practice be chosen to be sufficiently small so that the model remains stable. Note that the implementation of the coupling of the tension back to the string is not strictly speaking physically accurate. However, it has been found to be a suitable solution for sound synthesis purposes.

5. SIMULATIONS AND RESULTS

Parameter estimation procedures for the tension modulation model are described in [2, 7]. The nominal delay line lengths are determined by the desired nominal fundamental frequency. The reflection filters may be designed to give the best match to the frequency-dependent decay of the tone. The tension modulation parameter A may be derived from the string properties or match to a given initial amplitude of the pluck.

The TMDF output coupling coefficient g_{out} may be derived from recordings of instrument tones. The

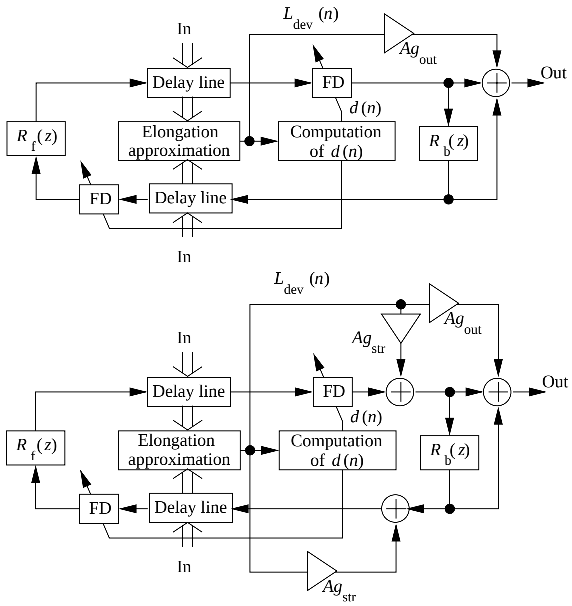

Figure 3: Dual-delay-line waveguide model of a string including TMDF output (top), and TMDF output and coupling (bottom).

technique is the following. The string is plucked at its midpoint so that the initial amplitude of the second harmonic mode in the string vibration is minimal. Frequency analysis is performed on a relatively short segment of the signal immediately after the attack. The relative levels of the first and second harmonic are used to determine the coupling since the second harmonic is mainly produced by nonlinear coupling from the first harmonic in the early part of the signal.

Unfortunately, the determination of the feedback coefficient g_{str} is not so straightforward unless the string vibration has been measured using, e.g., a magnetic pickup together with the radiated sound. If the two signals are available, the feedback coefficient g_{str} may be optimized to give the desired rise of the second harmonic in the midplucked tone. Fig. 4 shows three amplitude envelopes of the second harmonic of a synthetic tone with varying values of Ag_{str}. As seen in the figure, the rise of the amplitude is faster when the parameter is larger. Note that the curves have been placed a top of each other for visual inspection.

Fig. 5 shows the amplitude envelopes of the second harmonics detected in string vibration simulation and in the output signal (cf. the solid and the dashed lines in Fig. 2). The figure shows that the coupling of the tension modulation back to the string produces a similar raise in the amplitude envelope of the second harmonic in the synthetic signal as seen in the solid line. Again, the curves have been positioned for visual inspection.

Synthetic tones comparing different models are available at http://www.acoustics.hut.fi/˜ttolonen/icmc99/. The web page also describes the parameter values used in synthesizing the signals.

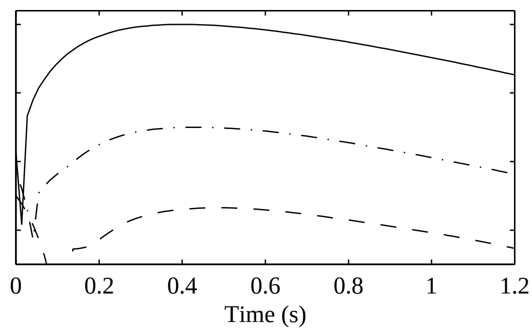

Figure 4: Amplitude envelopes of the second harmonic of the synthetic string vibration signal with $Ag_{\mathrm{str}} = 0.001$ (bottom curve), $Ag_{\mathrm{str}} = 0.002$ (middle curve), and $Ag_{\mathrm{str}} = 0.005$ (top curve).

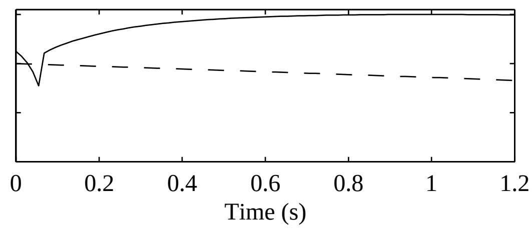

Figure 5: The amplitude envelopes of the second harmonic in the simulated string vibration (solid line) and in the output signal (dashed line).

6. SUMMARY AND CONCLUSIONS

In this paper a plucked-string model was presented that includes nonlinear tension modulation driving force coupling to the body and to the string. The proposed model was shown to reproduce the essential features caused by the TMDF coupling by comparing the behavior of harmonics in measured and synthesized signals. Sound examples of synthetic tones are available at http://www.acoustics.hut.fi/˜ttolonen/icmc99/.

The proposed model elaborates previous linear models and models that include the tension modulation phenomena. In several instruments, e.g. the kantele and the tanbur, the TMDF coupling is essential in order to properly simulate the behavior of the vibrating string.

7. ACKNOWLEDGMENT

This work has been financially supported by the GETA Graduate School, Tekniikan edistämissäätiö, Foundation of Jenny and Antti Wihuri, the Academy of Finland, CIMO, and Nokia Research Center.

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