1-D DIGITAL WAVEGUIDE MODELING FOR IMPROVED SOUND SYNTHESIS

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ABSTRACT

Digital waveguide modeling of one-dimensional vibrating structures in musical instruments is developed further in this paper based on finite difference time domain (FDTD) formulations. It is shown that such realizations can be developed, efficient enough to run in real time on modern processors and more flexible than the computationally less expensive commuted synthesis. FDTD waveguide structures are analyzed with lossless and lossy propagation, input and output ports, terminations and scattering junctions. The formation of static initial state of displacement wave from pointwise excitation in plucked string models is demonstrated, not possible with simple waveguides. Related sound synthesis examples can be found at `www.acoustics.hut.fi/demos/fdl`.

1. INTRODUCTION

Model-based sound synthesis of musical instruments is an active field of research where many useful algorithms for music and audio technology have been developed [1]. Digital waveguide models are proven an excellent methodology for developing computationally efficient yet physically plausible synthesis algorithms. Particularly 1-D wave propagation models can be made very efficient, the Karplus-Strong model being the extreme case [2, 3]. Commuted synthesis using digital waveguide principles [4, 5] is a more advanced, yet highly efficient technique for example for string instruments, where only the string is modeled and the rest, the plucking and the body response, are merged into a wavetable.

Commuted and consolidated waveguide synthesis, although capable of high-quality synthesis with minimal cost, has some limitations. Since the body and the pluck are sampled data, they cannot be controlled and modified parametrically as much as often desired. Rapidly increasing computational capacity of processors makes it feasible to use substantially more complex algorithms than before for real-time sound synthesis. Therefore it is well motivated to look forward to new formulations as far as they offer improved flexibility through parametric control, better realization of fine details, easier design and calibration of the models, or deeper understanding from instrument physics and handling viewpoint.

The motivation in this study was the observed limitations of the simple lossless waveguide, shown in Fig. 1. This model, consisting of two delay lines with terminations and input-output ports, simulates the traveling wave components as discretized from the d'Alembert solution of the wave equation [1]. For example, if the delay lines are lossless, there is no way to insert a proper triangular initial state of string displacement just locally, without taking into account more global information about the string system. It is necessary to distribute losses to the elements of the waveguide delay lines and to formulate the plucking force excitation properly.

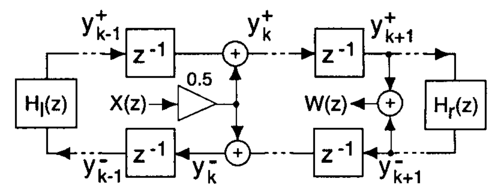

Fig. 1. Digital waveguide structure based on traveling wave components from d'Alembert solution of lossless wave equation.

The approach taken in this paper is the finite difference formulation of 1-D waveguides, seelar [7], [8], [9], but the present study is more towards efficient signal processing and its applications, although physical intuition is retained in the models. We formulate the 1-D finite difference time domain (FDTD) waveguide in terms of digital filtering and study its properties and controllability. The study is formulated particularly from the viewpoint of string instrument modeling.

2. THEORY OF 1-D FDTD WAVEGUIDES

In a one-dimensional lossless medium the wave equation is written

$$y_{tt} = c^2 y_{xx} \tag{1}$$

where y is (any) wave variable, subscript xx refers to second partial derivative in place variable x, tt second partial derivative in time t, and c is speed of wavefront in the medium of interest [6]. In a vibrating string we are primarily interested in transversal wave motion for which $c = \sqrt{T/\mu}$, where T is tension force and μ is mass per unit length of the string.

In order to discretize the wave equation for numeric modeling, the partial derivatives in (1) can be approximated by second order finite differences

$$y_{xx} \approx (2y_{x,t} - y_{x-\Delta x,t} - y_{x+\Delta x,t})/(\Delta x)^2 \tag{2}$$

$$y_{tt} \approx (2y_{x,t} - y_{x,t-\Delta t} - y_{x,t+\Delta t})/(\Delta t)^2 \tag{3}$$

By selecting discrete time sampling interval Δt to correspond to spatial sampling interval Δx, i.e., $\Delta t = c\Delta x$, and using index notation $k = x/\Delta x$ and $n = t/\Delta t$, Eqs. (2) and (3) result in

$$y_{k,n+1} = y_{k-1,n} + y_{k+1,n} - y_{k,n-1} \tag{4}$$

which is a special case of multidimensional digital waveguide as an FDTD formulation [10, 11]. From form (4) we can see that a new sample $y_{k,n+1}$ at position k and time index $n+1$ is computed as the sum of its neighbouring position values minus the value at the position itself one sample period earlier.

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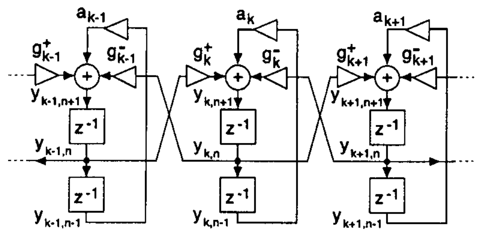

Fig. 2. Digital waveguide structure based on finite difference approximation of 1-dimensional wave equation.

Equation (4) can be interpreted as a spatio-temporal digital waveguide filter, shown in Fig. 2. The simple FDTD waveguide of Eq. (4) is realized by coefficients values $g_k^+ = 1$, $g_k^- = 1$ and $a_k = -1$. Further functionality with different values of these coefficients will be introduced below.

Insertion of a forward traveling impulsive wave component $u(n)$ into an FDTD waveguide can be realized by incrementing state variables by

$$y_{k,n} := y_{k,n} + 0.5u(n) \quad (5)$$

$$y_{k-1,n-1} := y_{k-1,n-1} + 0.5u(n) \quad (6)$$

and insertion in the backward traveling direction correspondingly

$$y_{k,n} := y_{k,n} + 0.5u(n) \quad (7)$$

$$y_{k+1,n-1} := y_{k+1,n-1} + 0.5u(n) \quad (8)$$

A wave impulse that starts traveling in both directions can be inserted as a superposition of these two directional components. Another useful way of exciting the FDTD waveguide is to insert

$$y_{k,n} := y_{k,n} + 0.5u(n) \quad (9)$$

$$y_{k+1,n} := y_{k+1,n} + 0.5u(n) \quad (10)$$

which starts a rectangular block function ('boxcar') of y_k to spread in both directions from the excitation point pair. This is different from exciting a series of impulses according to Eqs. (5)–(8), and is found applicable to form a proper initial state for pluck in a string.

Recursion formula (4) can be generalized to cover a variety of practical cases by extending it to

$$y_{k,n+1} = g_k^+ y_{k-1,n} + g_k^- y_{k+1,n} + a_k y_{k,n-1} \quad (11)$$

where coefficients g_{k-1}^+, g_{p+1}^-, and a_k can be used to create a variety of waveguide cases, examples of which are discussed next.

2.1. Lossy waveguide modeling

The finite difference approximation of a lossless homogeneous waveguide is an accurate replica of the corresponding analog one for bandlimited signals [1]. For lossy waveguides the situation is essentially more complex. The simplest way to accommodate propagation losses is to adjust the filter coefficients in Fig. 2 properly. Experimentally we have found the following guidelines

$$g^+ = g^- = 1 - d \quad (12)$$

$$a = 2bd - 1 \quad (13)$$

where d and b are loss factors. For a passive waveguide $0 \leq d \leq 1$ and $0 \leq b \leq 1$. For nearly frequency-independent loss of wave propagation, selection of b sligtly below 1, such as $b \approx 0.97$ when $d = 0.05$ or $b \approx 0.994$ when $d = 0.01$, is recommended. Figure 3a depicts the magnitude spectrum measured in a point 100 steps from the excitation point for the former case. More control over frequency-dependency of losses requires higher order filtering per waveguide section, but in many cases this is not necessary since frequency-dependent losses can be approximated by designing a proper termination of the FDTD waveguide.

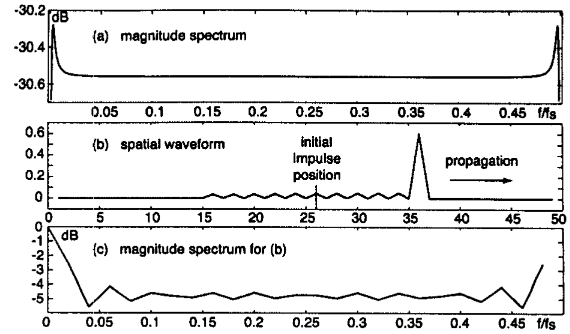

Fig. 3. Lossy FDTD waveguide propagation: (a) damping for $d = 0.05$ and $b = 0.97$, response after 100 steps propagation; (b) frequency scales are normalized by sampling rate.

If $b = 1.0$, the impulse response becomes almost unstable, showing ripple or an 'afterhump', a combination of very low and high frequency oscillation that only gradually decays off [1]. Fig. 3b shows a spatial snapshot of an impulse that travelled 10 steps for $d = 0.05$ and $b = 1.0$. The magnitude spectrum in Fig. 3c of this spatial signal reveals a symmetry point, typical to FDTD models, at half of the Nyquist frequency [1].

2.2. Termination and scattering (lossless and lossy)

Simple terminations of an FDTD waveguide can be realized for example in the following way. Phase-inverting reflection, such as displacement at a fixed end-point of a string, is realized by keeping $y_{K,n+1} := 0$ at the terminating node K. A non-inverting termination is achieved by $y_{K,n+1} := 2y_{K-1,n} - y_{K,n-1}$. Impedance match with no reflection is implemented by $y_{K,n+1} := y_{K-1,n}$.

A generalization of termination is a scattering junction. For a lossless impedance discontinuity (Kelly-Lochbaum junction), reflection coefficient r_k is defined as $r_k = (R_k^+ - R_k^-)/(R_k^+ + R_k^-)$, where R_k^+ and R_k^- are the wave impedances to the right and left of the junction, respectively. In such a case the filter coefficients in Fig. 2 can be easily derived:

$$g_k^+ = 1 + r_k, \quad g_k^- = 1 - r_k, \quad \text{and} \quad a_k = -1 \quad (14)$$

For a resistively loaded (by R_k^*) junction with wave impedance conditions R_k^+ and R_k^- defined above the filter coefficients will be

$$g_k^+ = 2/(1 + R_k^-/R_k^* + R_k^-/R_k^+) \quad (15)$$

$$g_k^- = 2/(1 + R_k^+/R_k^* + R_k^+/R_k^-) \quad (16)$$

$$a_k = b(2 - g_k^+ - g_k^-) - 1 \quad (17)$$

Equation (17) is formed from (13) by setting $d = (g_k^+ + g_k^-)/2$, which is found useful experimentally as was the case with (13).

[1] In fact, if $b = 1$, propagation is of band-reject type where the DC and Nyquist band energies in the line do not attenuate.

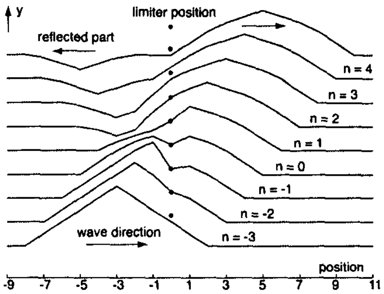

Fig. 4. Right-traveling triangular wave, hard-limited at the point marked by a dot. For $n = [-2, 2]$ string touches the constraint.

In string modeling, the equations above can be used for example to approximate the effect of bridge impedance and coupling of strings via it (extendable to a port of multiple strings [1]), attached loads such as a touching finger or plectrum, or any linearizable loading. If the loading impedance R_k^* is not resistive, coefficients g_k^+, g_k^-, and a_k^* will become filters instead of scalar coefficients, which is a more demanding task to design, not discussed here.

2.3. Nonlinear couplings

There are many nonlinear phenomena in musical instruments where digital waveguides need to be extended with corresponding simulation algorithms. In string modeling, bow-string interaction in the violin and tension modulation in some instruments are examples thereof. Strings hitting frets or the fingerboard is a common event in slap base playing [13] or in flamenco guitar playing. This means that the vibration displacement is hard-limited.

In the present framework the simulation of such limiting nonlinearity is conceptually easy since displacement can be used explicitly as wave variable $y_{k,n}$ (Fig. 2). If $y_{k,\mathrm{lim}}$ is the constrained value for displacement, the following rule can be used to simulate hard limitation at point k. After computing a new value of $y_{k,n+1}$ normally as in Eq. (11), a check is done if $y_{k,n+1} > y_{k,\mathrm{lim}}$. If true, an impulse excitation with amplitude $\Delta y_k = -(y_{k,n+1} - y_{k,\mathrm{lim}})$ is inserted into the node by rule of Eqs. (5)–(8) to keep the displacement within the limit value. Figure 4 illustrates a simulated case of a triangular wave hitting such a constraint.

2.4. Fractionally positioned terminations

Adjusting the effective length of a digital waveguide in fractional delay units is a required property in order to tune the fundamental frequency accurately. In [14], interpolation and deinterpolation by Lagrance interpolation techniques was proposed and methods using allpass interpolators were discussed. In the present FDTD waveguides the most natural solution is to design a fractionally positioned scattering junction. As a simple special case of that, linearly interpolated fractional termination is analyzed. If

$$g_k^+ = 1 - \delta, \quad g_k^- = 1 + \delta, \quad \text{and} \quad a_k = -2.0 \qquad (18)$$

then the effective reflection point at low frequencies is at $k + \delta$. Due to traveling the fractional part δ twice, the backward reflecting component corresponds to forward signal

$$y_{k,\mathrm{refl}} = (1 - \delta) y_k^+ + \delta\, y_{k-2}^+ \qquad (19)$$

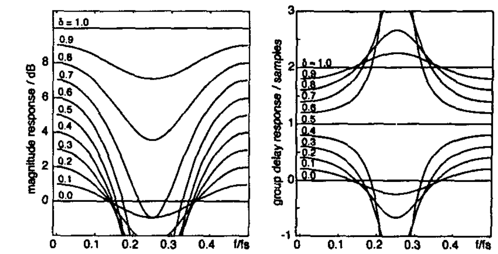

Fig. 5. Reflection at a fractional point termination: (a) magnitude and (b) group delay response, where δ is the fraction parameter.

This corresponds to first-order Lagrange interpolation, but between samples at points k and $k + 2$, which makes it deviate from ideal fractional delay at high frequencies more than in interpolation between adjacent samples. It also mirrors the response with symmtry point at one fourth of sampling rate. Figure 5 depicts the magnitude and group delay of the reflection for values of δ parameter. Another detail to be taken into account is that there is a forward traveling wave component passing the termination point. It should be terminated completely later along the waveguide not to reflect and return to the computation of desired signal samples.

Since the technique proposed here is useful only with high sampling rates or high losses at high frequencies, better fractional delay terminations and scattering port techniques remain to be developed.

2.5. Initial state formation and plucking of a string

One of the problems with lossless consolidated and commuted digital waveguide synthesis is the formation of proper initial state within the waveguide for example just prior to a pluck. Only if the wave variable is acceleration the excitation can be injected locally since the ideal pluck is an impulse [12]. This is one reason why in string models acceleration is often applied. For velocity or displacement the formation of natural initial state corresponding to a triangular displacement shape and zero velocity cannot be injected locally, without global knowledge of string properties. This is easy to understand since in a lossless digital waveguide a triangularly shaped static displacement is impossible.

In the FDTD waveguide of Fig. 2 the formation of initial state is possible if proper excitation is applied and damping is controlled by filter parameters, g's and a's. Resembling the finite difference formulation in [8] we found by experimentation that when parameter b in Eq. (13) is given value 1 and applying proper damping factor $d < 1$, the string developes a triangular initial shape if a step function of force is inserted to the plucking point of the string by Eqs. (9)–(10).

Figure 6 depicts simulated formation of initial state in an FDTD waveguide having length of 100 unit delays with force applied to position 35 in two cases: (a) high loss ($a_k = -0.96$ and $g_k^+ = g_k^- = 0.98$) and (b) low loss ($a_k = -0.998$ and $g_k^+ = g_k^- = 0.996$). In the former case the triangular state results rapidly without much oscillation, but in the latter case the string still oscillates strongly after 300 samples of computation. In both cases terminations are lossless (inverting) and string losses are distributed uniformly along the waveguide, but for better simulation of reality, more damping can be applied at the position of pluck than elsewhere by setting lower values of a's and g's locally.

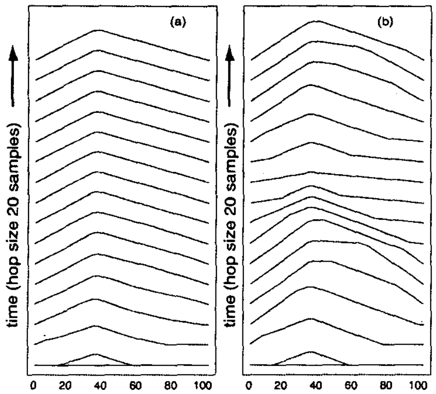

Fig. 6. Initial state formation in a string with (a) high loss and (b) low loss damping. Horizontal axis = string position, vertical axis = string displacement, time grows upwards. Excitation (force step function) at position 35.

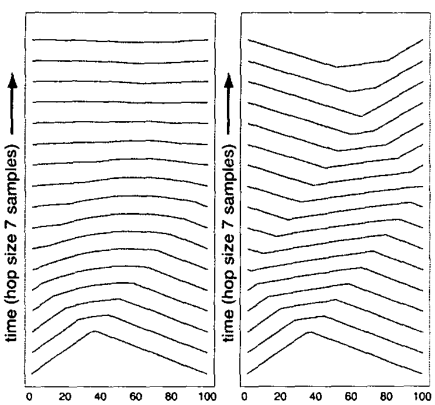

Fig. 7. Vibration of string after release from initial state with (a) high loss and (b) low loss damping. Horizontal axis = string position, vertical axis = string displacement, time grows upwards.

When the string is released at the moment of plucking, it starts to oscillate autonomously. One half of the initial displacement shape starts to travel backward and another half forward in the waveguide. Depending on damping coefficient values the vibration decays sooner or later. Figure 7 depicts the vibration in two cases: (a) high loss damping ($a_k = -0.96$ and $g_k^+ = g_k^- = 0.98$) and (b) low loss damping ($a_k = -0.996$ and $g_k^+ = g_k^- = 0.998$).

3. DISCUSSION AND CONCLUSIONS

This paper has introduced powerful features of 1-D finite difference waveguide models with distributed losses, scattering junctions including fractionally positioned terminations, and proper initial state formation. A sophisticated dual-polarization guitar string model can be shown to run easily in real time on a modern computer, such as a 500 MHz PowerPC processor.

FDTD waveguide modeling, closer to physical reality than consolidated and commuted digital waveguide synthesis, reveals a wide range of further problems to be solved and applications to be developed. The mathematical basis, partly experimentally explored here, should be related to the physics of such systems. Dispersive waveguides are to be added and passive nonlinearities, including shock waves in tubes and tension modulation in strings, need proper experimentation. Detailed interaction between finger/plectrum and string in plucking, bow-string interaction in the violin, sympathetic coupling through bridge admittance, etc., are examples of problems in string modeling where general principles have already been known, but new formulations of digital waveguides allow for improved techniques to explore them in detail and implement practical model-based synthesis algorithms.

Examples of string instrument sounds by FDTD models can be found at `www.acoustics.hut.fi/demos/fdl`.

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