Robust Hybrid Beamforming for Integrated Sensing and Communications via Learned Optimization

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Motivation

- **Integrated sensing and communications (ISAC)** has attracted enormous research interest from both academia and industry.
- Deep learning has been widely applied to beamforming designs recently. The black-box nature makes DNNs hard to deploy in practical scenarios [**?**].
- To eliminate this issue, model-based learning framework (e.g., **algorithm unrolling**) has been proposed to make neural networks interpretable [**?**].
- **Steering vector mismatches** arise in practice because of imperfections in array calibration, distorted antenna array shape, etc [**?**].

This work:

Robust hybrid beamforming for ISAC system under bounded uncertainties in sensing reception using algorithm unrolling

Introduction to Algorithm Unrolling (AU)

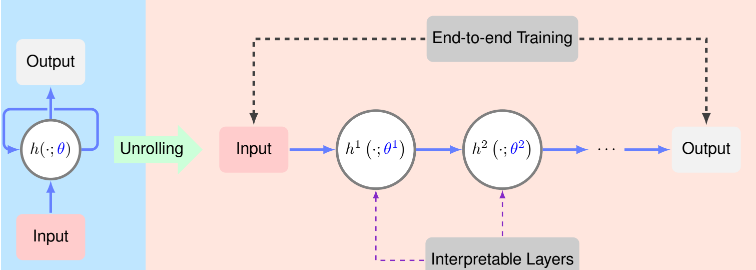


Figure from [?]

Procedure of AU

- pick an iterative algorithm
- unroll it to a neural network (NN)
- select a set of NN parameters to learn

Advantages of AU

- (can) achieve better performance
- naturally inherit interpretability
- require fewer training data

System Model

System composition

- A dual-function radar-communication base station (BS)
  N_t transmit antennas, N_r receive antennas
- M single-antenna communication users
- T point-like targets
- J interferences

Transmit signal

$$\mathbf{x} = \mathbf{F}_{\mathrm{a}}\mathbf{F}_{\mathrm{d}}^{\mathrm{c}}\mathbf{c} + \mathbf{F}_{\mathrm{a}}\mathbf{F}_{\mathrm{d}}^{\mathrm{s}}\mathbf{s}$$

where

- $\mathbf{F}_{\mathrm{a}} \in \mathbb{C}^{N_t \times L}$: analog beamforming matrix (L is # of RF chains)
- $\mathbf{F}_{\mathrm{d}}^{\mathrm{s}} \in \mathbb{C}^{L \times T}$: digital beamforming matrix for sensing
- $\mathbf{F}_{\mathrm{d}}^{\mathrm{c}} \in \mathbb{C}^{L \times M}$: digital beamforming matrix for communications
- $\mathbf{c} \in \mathbb{C}^{M \times 1}$: communication data symbols satisfying $\mathbf{c} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$
- $\mathbf{s} \in \mathbb{C}^{T \times 1}$: radar waveforms satisfying $\mathbb{E}[\mathbf{s}\mathbf{s}^{\mathsf{H}}] = \mathbf{I}_T$

Communication performance metric

Achievable sum-rate (SR):

$$\gamma_{\mathrm{c}} = \sum_{m=1}^{M} \log(1 + \mathsf{SINR}_m^{\mathrm{c}}),$$

where

$$\mathsf{SINR}_m^{\mathrm{c}} = \frac{|\mathbf{h}_m^{\mathsf{H}} \mathbf{F}_{\mathrm{a}} \mathbf{f}_{\mathrm{d},m}|^2}{\sum_{j=1,\, j\neq m}^{M+T} |\mathbf{h}_m^{\mathsf{H}} \mathbf{F}_{\mathrm{a}} \mathbf{f}_{\mathrm{d},j}|^2 + \sigma_m^2}$$

σ_m^2 = noise power at m^{th} user equipment

$\mathbf{h}_m$ = communication channel from BS to the m^{th} user ($\in \mathbb{C}^{N_t \times 1}$)

Sensing performance metric

At BS, t^{th} target's echo signal is filtered by a **receive combiner** $\mathbf{w}_t \in \mathbb{C}^{N_r \times 1}$ to obtain y_t^s.

Mutual information (MI) for multiple (T) targets:

$$\gamma_{\mathrm{s}} = \sum_{t=1}^{T} \log(1 + \mathsf{SINR}_t^{\mathrm{s}}),$$

where the **sensing SINR** for the t^{th} target:

$$\mathsf{SINR}_t^{\mathrm{s}} = \frac{\|\mathbf{w}_t^{\mathsf{H}} \mathbf{H}_t^{\mathrm{s}} \mathbf{F}_{\mathrm{a}} \mathbf{F}_{\mathrm{d}}\|_2^2}{\sum_{p=1,\, p\neq t}^{T} \|\mathbf{w}_t^{\mathsf{H}} \mathbf{H}_p^{\mathrm{s}} \mathbf{F}_{\mathrm{a}} \mathbf{F}_{\mathrm{d}}\|_2^2 + \sum_{j=1}^{J} \|\mathbf{w}_t^{\mathsf{H}} \mathbf{H}_j^{\mathrm{s}} \mathbf{F}_{\mathrm{a}} \mathbf{F}_{\mathrm{d}}\|_2^2 + \sigma_t^2 \|\mathbf{w}_t\|_2^2},$$

$\mathbf{H}_l^{\mathrm{s}} = \alpha_l \mathbf{a}_{\mathrm{r}}(\theta_l) \mathbf{a}_{\mathrm{t}}^{\top}(\theta_l)$ is the target steering matrix (radar round-trip channel)

$\theta_l =$ direction of the point target

$\alpha_l =$ propagation reflection coefficients $(\sim \mathcal{N}(0, \beta_l))$

System Model

- **Uncertainty model**

$$\mathbf{a}_{\mathrm{r}} = \hat{\mathbf{a}}_{\mathrm{r}} + \boldsymbol{\delta}_{\mathrm{r}},$$

where $\hat{\mathbf{a}}_{\mathrm{r}}$ = presumed receive steering vector, $\boldsymbol{\delta}_{\mathrm{r}}$ = error bounded by ε_r:

$$\boldsymbol{\delta}_{\mathrm{r}} \in \mathcal{A}_{\mathrm{r}} \triangleq \left\{ \boldsymbol{\delta}_{\mathrm{r}} \mid \frac{1}{N_r} \|\boldsymbol{\delta}_{\mathrm{r}}\|_2 \leq \varepsilon_r \right\}.$$

- **Problem** (maximize the communication SR and worst-case sensing MI over $\mathcal{A}_{\mathrm{r}}$):

$$\begin{aligned} \max_{\mathbf{F}_{\mathrm{a}}, \mathbf{F}_{\mathrm{d}}, \mathbf{w}_t} \quad & \left(\rho \gamma_{\mathrm{c}} + (1-\rho) \min_{\boldsymbol{\delta}_{\mathrm{r}} \in \mathcal{A}_{\mathrm{r}}} \gamma_{\mathrm{s}} \right) \\ \text{s.t.} \quad & |[\mathbf{F}_{\mathrm{a}}]_{m,n}| = 1, \\ & \|\mathbf{w}_t\|_2^2 = 1, \\ & \|\mathbf{F}_{\mathrm{a}} \mathbf{F}_{\mathrm{d}}\|_{\mathrm{F}}^2 = P_t, \end{aligned}$$

where ρ denotes fixed weight, and P_t is the power budget of BS.

Projected Gradient Descent and Ascent (PGDA)

PGD-based minimization over the uncertainty set $\mathcal{A}_{\rm r}$

- Gradient step

$$\tilde{\boldsymbol{\delta}}_{\rm r}^{(i+1)} = \boldsymbol{\delta}_{\rm r}^{(i)} - \mu_{\rm r}^{(i)} \frac{\partial \gamma_{\rm s}\left(\mathbf{F}_{\rm a}^{(i)}, \mathbf{F}_{\rm d}^{(i)}, \mathbf{w}_t^{(i)}, \boldsymbol{\delta}_{\rm r}^{(i)}\right)}{\partial \boldsymbol{\delta}_{\rm r}^{*}},$$

where $\mu_{\rm r}$ is the step size for $\boldsymbol{\delta}_{\rm r}$.

- Projection step

$$\boldsymbol{\delta}_{\rm r}^{(i+1)} = \min\left\{\frac{\varepsilon_r N_r}{\|\tilde{\boldsymbol{\delta}}_{\rm r}^{(i+1)}\|_2}, 1\right\} \tilde{\boldsymbol{\delta}}_{\rm r}^{(i+1)}.$$

Projected Gradient Descent and Ascent (PGDA)

PGA-based maximization over $\mathbf{F}_\mathrm{a}$, $\mathbf{F}_\mathrm{d}$, and $\mathbf{w}_t$

- Gradient step ($\gamma = \rho\gamma_\mathrm{c} + (1-\rho)\gamma_\mathrm{s}$)

$$\tilde{\mathbf{F}}_\mathrm{a}^{(i+1)} = \mathbf{F}_\mathrm{a}^{(i)} + \mu_\mathrm{a}^{(i)} \frac{\partial\gamma\left(\mathbf{F}_\mathrm{a}^{(i)}, \mathbf{F}_\mathrm{d}^{(i)}, \mathbf{w}_t^{(i)}, \boldsymbol{\delta}_\mathrm{r}^{(i+1)}\right)}{\partial\mathbf{F}_\mathrm{a}^*},$$

$$\tilde{\mathbf{F}}_\mathrm{d}^{(i+1)} = \mathbf{F}_\mathrm{d}^{(i)} + \mu_\mathrm{d}^{(i)} \frac{\partial\gamma\left(\mathbf{F}_\mathrm{a}^{(i)}, \mathbf{F}_\mathrm{d}^{(i)}, \mathbf{w}_t^{(i)}, \boldsymbol{\delta}_\mathrm{r}^{(i+1)}\right)}{\partial\mathbf{F}_\mathrm{d}^*},$$

$$\tilde{\mathbf{w}}_t^{(i+1)} = \mathbf{w}_t^{(i)} + \mu_\mathrm{w}^{(i)} \frac{\partial\gamma\left(\mathbf{F}_\mathrm{a}^{(i)}, \mathbf{F}_\mathrm{d}^{(i)}, \mathbf{w}_t^{(i)}, \boldsymbol{\delta}_\mathrm{r}^{(i+1)}\right)}{\partial\mathbf{w}_t^*},$$

where μ_a, μ_d, and μ_w are the step sizes for $\mathbf{F}_\mathrm{a}$, $\mathbf{F}_\mathrm{d}$, and $\mathbf{w}_t$, respectively.

- Projection: $[\mathbf{F}_\mathrm{a}^{(i+1)}]_{m,n} = \frac{[\tilde{\mathbf{F}}_\mathrm{a}^{(i+1)}]_{m,n}}{|[\tilde{\mathbf{F}}_\mathrm{a}^{(i+1)}]_{m,n}|}$, $\mathbf{F}_\mathrm{d}^{(i+1)} = \frac{\sqrt{P_t}\tilde{\mathbf{F}}_\mathrm{d}^{(i+1)}}{\left\|\mathbf{F}_\mathrm{a}^{(i+1)}\tilde{\mathbf{F}}_\mathrm{d}^{(i+1)}\right\|_\mathrm{F}}$, $\mathbf{w}_t^{(i+1)} = \frac{\tilde{\mathbf{w}}_t^{(i+1)}}{\|\tilde{\mathbf{w}}_t^{(i+1)}\|_2}$.

Unrolled PGDA

- **Aim**: use algorithm unrolling to tune step sizes in PGDA algorithm based on data.
- **Trainable parameters**:

$$\mathbf{\Theta} \triangleq [\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \ldots, \boldsymbol{\mu}_I]$$

  with $\boldsymbol{\mu}_i \triangleq [\mu_{\mathrm{a}}^{(i)}, \mu_{\mathrm{d}}^{(i)}, \mu_{\mathrm{w}}^{(i)}, \mu_{\mathrm{r}}^{(i)}]^\top$ = step size vector for i^{th} iteration.
- **Loss function** ($\gamma = \rho\gamma_{\mathrm{c}} + (1-\rho)\gamma_{\mathrm{s}}$):

$$\mathcal{L}(\mathbf{\Theta}) = -\frac{1}{|\mathcal{D}|}\sum_{d=1}^{|\mathcal{D}|}\frac{1}{I}\sum_{i=1}^{I}\log(1+i)\gamma(\mathbf{h}_m^{(d)}, \mathbf{H}_l^{\mathrm{s},(d)}, \mathbf{F}_{\mathrm{a}}^{(i)}, \mathbf{F}_{\mathrm{d}}^{(i)}, \mathbf{w}_t^{(i)}, \boldsymbol{\delta}_{\mathrm{r}}^{(i)}),$$

  - weighted sum of losses (enhancing the system to improves along the iterations [?]).
  - $\mathcal{D}$ = data set containing communication and sensing channel realizations.
- **Tune** the hyperparameter matrix:

$$\mathbf{\Theta}^\star = \arg\min_{\mathbf{\Theta}} \mathcal{L}(\mathbf{\Theta})$$

Numerical Evaluation

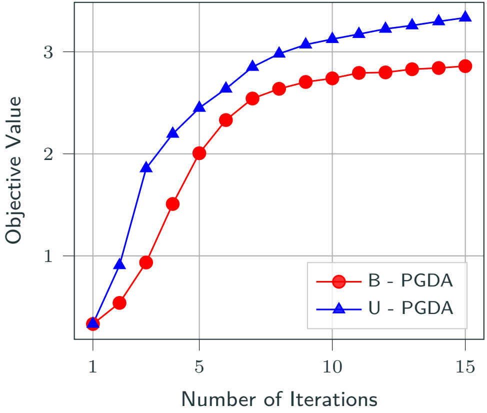
Parameter settings

- $M = 4,\ T = 8,\ J = 4,\ N_t = 6,\ N_r = 8,\ L = 4$
- $P_t = 5$ dB (power budget)
- weight factor $\rho = 0.8 \quad (\gamma = 0.8\gamma_\mathrm{c} + 0.2\gamma_\mathrm{s})$

Training settings:

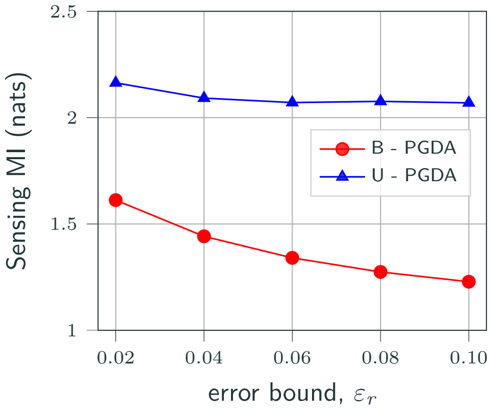
- Channel vector $\mathbf{h}$, reflection coefficients α_t (for target) and α_j (interfer) are circular Gaussian noises with variances 0 dB , $\beta_t = 2$ dB and $\beta_j = 3$ dB, respectively
- Dataset sizes for training and testing: 1000 and 100
- SGD optimizer with learning rate of 0.01
- Fixed step size for PGDA and initial step size for unrolled PGDA: 0.1

Convergence evaluation



- The benchmark PGDA may easily get trapped in local optima when solving non-convex problem.
- The unrolled PGDA is capable of **learning the update rules from data**, allowing it to fit the objective function and escape local optima more effectively than the benchmark.

Robustness evaluation



- The sensing MI curve achieved by the unrolled PGDA has lower slope than that corresponding to the benchmark PGDA.
- The unrolled PGDA is less sensitive than the benchmark PGDA in the face of increased level of uncertainties.
- The unrolled PGDA is more robust than the benchmark PGDA.

Conclusion

- We formulated an optimization problem aimed at maximizing the communication SR and worst-case sensing MI with bounded uncertainty.
- A PGDA algorithm was developed for solving the formulated optimization problem.
- Unrolled PGDA algorithm was proposed, and step sizes tuned based on the data.
- unrolled PGDA exhibits **faster convergence** and **better robustness** than the PGDA

Future work (in progress):

- The uncertainties of communication channel and transmit steering vector will be explored.
- A "safeguard" mechanism could be explored to ensure algorithm unrolling performs no worse than traditional algorithms on out-of-distribution tasks.

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